Supporting 1st Grade students with difficulties in Mathematics.

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CONTENTS

Introduction

In primary school, a large portion of mathematics instruction is devoted to the study of whole numbers and operations with whole numbers. A number, such as number five, is a concept that represents a certain quantity. It may not be obvious to many students that the number is the outcome of an abstract procedure. We frequently use numbers to indicate sets of discrete items in both life and mathematics. The combination of all of this and pattern teaching helps students improve their mathematical abilities. Recent studies have demonstrated, in particular, that pattern formation and pattern repetition abilities are distinct predictors of later mathematical ability (Lüken et al., 2014; Nguyen et al., 2016; Rittle-Johnson et al., 2017).

Specifically, students at the 1st grade of primary school are expected to develop the following core knowledge and skills (National Council of Teachers of Mathematics, 2000).:

- **Recognizing, describing, and extending pictorial patterns**
- **Reading, recognizing, comparing, ordering, and representing whole numbers up to 10**
- **Modeling mathematical stories of addition and subtraction with appropriate mathematical sentences**
- **Adding and subtracting up to 10 fluently (e.g., 3+2, 5+4, 8+2, 7-4, 10-6)**
- **Solving simple additive structure problems (grouping, changing, comparing)**

Students may face difficulties in developing understanding about the concept of numbers even from their early experiences with mathematics (Dowker, 2005). Hence, they need help and support to overcome these obstacles.

The difficulties faced by some children in terms of understanding and mastering basic mathematical concepts and procedures in the 1st Grade can be addressed through remedial teaching courses that take place either individually or within small groups of children. These lessons need to be differentiated from the lessons taught to the whole class so that they address the difficulties faced by these students. To organize and implement this kind of lessons, the following three basic parameters need to be considered: (a) the students' characteristics, (b) the teaching methodology, and (c) the teaching activities (Dowker, 2005).

STUDENTS’ CHARACTERISTICS

In the 1st grade, children are expected to explore and master the aforementioned basic mathematical concepts and procedures through systematic teaching. The incomplete understanding of these topics in Grade 1 creates difficulties with respect to the smooth development of children's mathematical knowledge in later grades and increases the risk of functional illiteracy in mathematics. Many studies have revealed that 1st graders who struggle to develop mathematical concepts exhibit behaviors such as:

- They enumerate quantities slowly. They have difficulty in enumerating quantities based on groupings (e.g., 2-2).
- They have difficulty in writing numbers correctly. They write at a slow pace.
- They do not easily associate the symbolic form of a number with the quantity it represents.
- They do not fluently recall the result of adding or subtracting numbers up to 10 (e.g., 5+3, 10-8).
- They do not follow procedures or rules.
- When performing simple calculations, they make mistakes or work very slowly because they rely on the enumeration of quantities (e.g., they calculate the sum 5+3 by counting all 1, 2, 3, 4, 5... 6, 7 ,8).
- They do not understand and remember basic math vocabulary (e.g., addition, subtraction, sum, difference).
- They have difficulty in understanding simple word problems. They forget what their goal is and what they are doing in the middle of a process or during problem solving.
- They have difficulty in interpreting and constructing representations.
- They have difficulty in applying procedures that involve multiple steps.

2. TEACHING METHODOLOGY

2.1. Introduction

An important parameter on which remedial teaching needs to be based is the teaching methodology, that is, how teachers deliver these courses. Teachers should be able to properly utilize the complementary material in combination with appropriate practices, in order to reach the expected outcomes. Mathematics lessons aim to the development of quantitative and abstract thinking among students. To achieve this aim, it is important to present to students’ various representations of mathematical concepts and procedures, The development of quantitative thinking can be facilitated using real objects and manipulatives, such as pictures and diagrams that allow the visualization of quantities. The development of abstract thinking lies in the use of mathematical symbols and language. Connections need to be made between quantities and symbols. Students with difficulties may not be able to connect the symbolic form of a number with the quantity it represents. This is why the teaching material needs to be enriched with multiple representations of numbers.

2.2 Levels of Teaching

To deal with such difficulties, it is important that teaching involves three basic levels: the concrete, the visual, and the symbolic level. The concrete level allows children to represent mathematical concepts and processes quantitatively using three-dimensional objects, thus achieving understanding through tangible, kinesthetic experiences. After students have mastered the concrete level, they work at the visual level that refers to the use of visual representations. In particular, understanding of concepts and relationships is enhanced using diagrams and pictures. After the visual level, students need to engage with the symbolic level, where concepts and processes are represented using mathematical symbols to represent and model mathematical situations, such as "+, -, =".

The three levels must be used in the teaching of each mathematical unit, so that the students develop a comprehensive understanding of each concept. For example, teaching number composition and decomposition might begin with exploration using cubes, then practice through picture-matching exercises, and finally representation with addition math sentences.

2.3 Learning Objectives

A successful lesson relies on students' active engagement with the tasks. To achieve this goal, specific learning objectives must be set by the teachers and relevant teaching material must be used. For example, if our goal is to count quantities from 1 to 5, the material should include activities that are exclusively related to the process of counting and not to other processes, such as writing or representing numbers up to 5.

The challenges described above must be taken into consideration while creating a series of lessons, particularly those that attempt to engage students who lack of concentration or students that get easily distracted. The content of each lesson needs to be precise to support teachers in helping these students to be focused and gain better control over what they learn and what they are expected to perform. As a result, it is critical that each lesson —or group of lessons —establishes a clear learning objective for a particular topic. The teacher should essentially avoid trying to cover more than one topic within a lesson. It is also important to keep in mind that the number of lessons devoted to a particular aim depends on the progress of the student who has certain difficulties. That is, a single lesson or more may be dedicated, depending on the student's response. Thus, the lessons come to a closure once the initial objective has been met. For example, several lessons are devoted to the idea of patterns and the method through which students recognize and describe patterns. The appropriate objective-related lessons will be finished once students have sufficiently mastered it. The next cycle of lessons that can focus on a new idea, i.e., pattern extension, can then be initiated.

Another important aspect of these lessons is conducting regularly brief formative assessments that assess specific learning goals during a lesson. In this way, the teacher gets a clear picture of each student's current knowledge and abilities and the areas on which the student needs to be improved.

As a result, the lessons that will follow the formative assessment will be adapted based on the results of the formative assessment that reflect the students' level and progress.

2.4 Use of examples from the child's familiar environment

For mathematics to be meaningful, it is critical the mathematical concepts and procedure to be presented through situations, objects, and words that children are familiar with and encounter in their daily life. To allow the students to concentrate on the mathematical idea under investigation and build meaning, we should not interfere with students' learning by introducing other ideas that the students may not be familiar with.

2.5 Multiple Examples

The ability to replicate and implement multi-step procedures is one of the challenges faced by pupils. Children may lose track of the process objective and the specific steps of what they are doing in the middle of a process. Therefore, teachers need to use multiple examples when students are learning mathematical procedures. That is, to present them with many solved examples and then ask them to apply the procedure on their own. Meanwhile, we also need to provide our own examples on the board or use manipulatives and/or digital tools.

2.6 Scaffolding Questions

The teacher is encouraged to use appropriate questioning techniques that focus on both the processes and the underlying reasoning. This is crucial for students with difficulties, who find it challenging to memorize a list of steps, especially those that do not make sense. To direct their attention and encourage reflection on the actions being taken, students must be scaffolded via supportive questions that enhance their conceptual understanding. So, for students to work through an example, they need guidance through targeted, helpful questions.

Students should be able to explain the aim in each lesson and reflect on their learning at the end of each lesson. This routine is facilitated when teachers provide students with the opportunity to "think out loud."

For example, when studying the addition strategy "count on",

the teacher can elaborate on a specific example
and ask the following questions:

- ✓ *Why did the girl write 2+3=3+2?*
- ✓ *Is her way of thinking, correct?*
- ✓ *How will the calculation of 3+2 help her, instead of 2+3?*
- ✓ *Why does the girl start counting from 3?*
- ✓ *Do you agree with her thinking?*

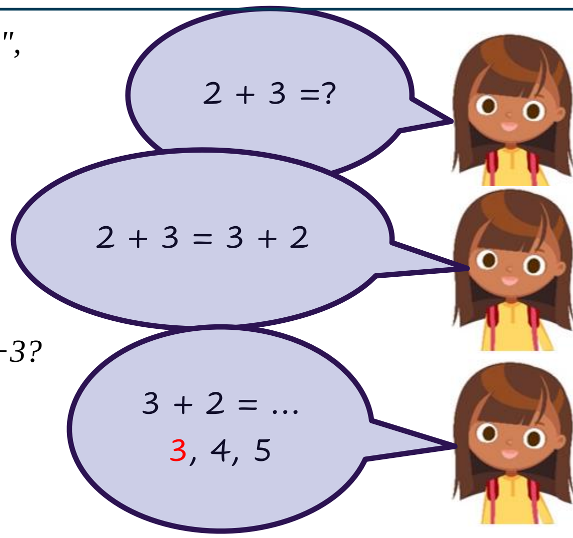


Figure 2.1: Counting on in addition.

2.7 Mathematical tools and technology

An idea or procedure should be taught in a variety of ways for students to fully understand it. Different representations, both quantitative and symbolic, are necessary to support the child's efforts to provide meaning to mathematical symbols, especially those children who find it difficult to relate the symbolic form of a mathematical idea to the quantity or quantitative relations it represents. Therefore, it is essential to use various manipulatives to represent mathematical procedures.

For example, in lessons dealing composition and decomposition of numbers up to 10, it is recommended to use UNIFIX cubes, to analyze a number in different ways. That is, a paper and pencil exercise that refers, for example, to the composition and decomposition of number 4, can be combined with the use of cubes, so that children see that the quantity 4 has different additive structures, i.e., 3 and 1 or 2 and 2 or 0 and 4 etc.

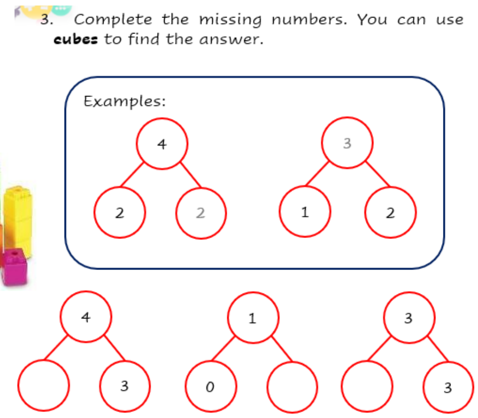
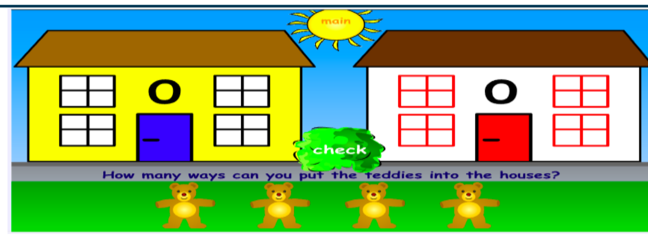


Figure 2.2: Decomposition of numbers with unifix cubes.

The use of digital tools has an added value when used for the introduction of a concept, the presentation of a process, and/or for further practice of taught concepts.

For example, for the composition and decomposition of numbers up to 10, there are digital applications that enable the decomposition of a quantity of objects into two sub-quantities in all possible ways.



https://www.topmarks.co.uk/Flash.aspx?f=WaystoMake

Figure 2.3: Decomposition of numbers with applications.

2.8 Mathematical Terminology

One of the main challenges that students may face is acquiring vocabulary related to basic mathematical concepts and procedures. It is recommended to write down important mathematical terms (basic mathematical vocabulary) on the whiteboard in the classroom so that the students can refer to them during learning and practicing.

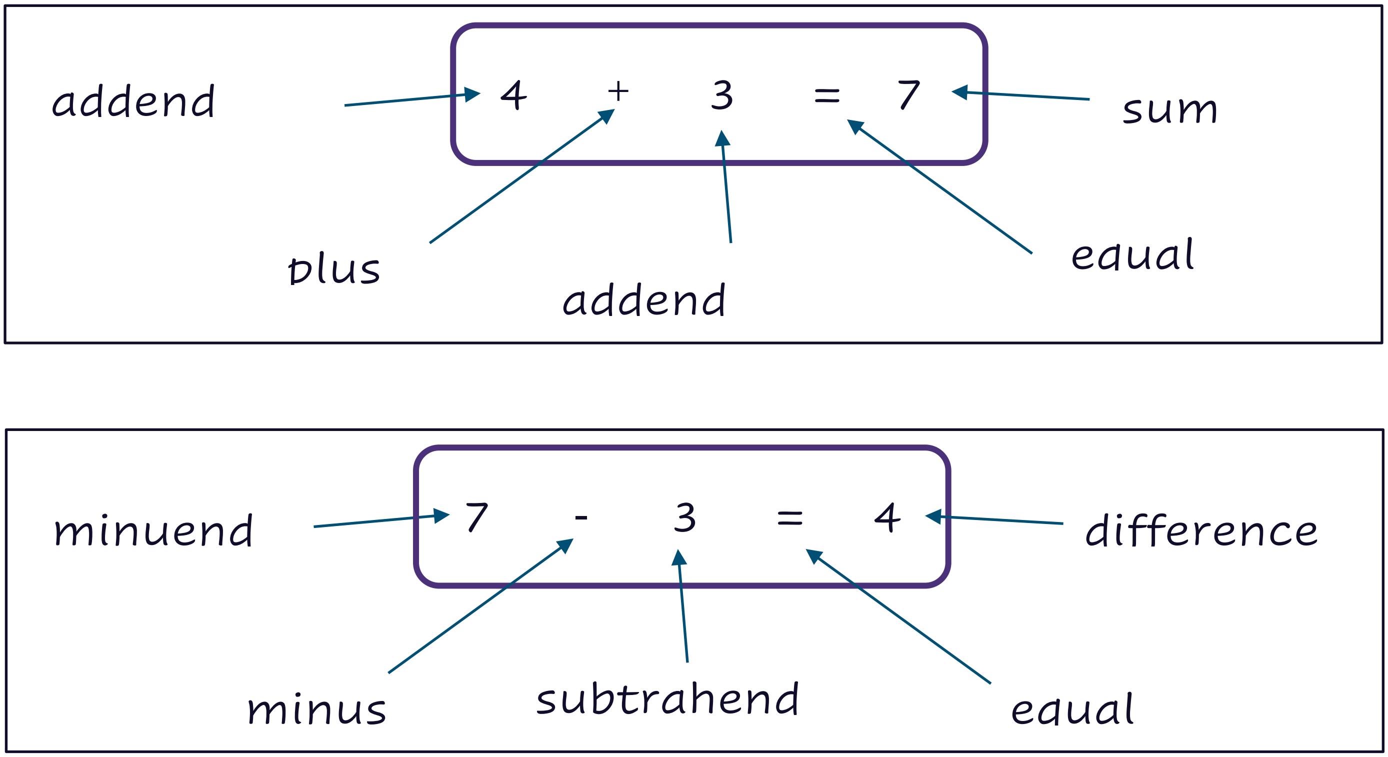


Figure 2.4: Mathematical terminology in addition and subtraction.

2.9 Systematic Repetition

A difficulty that does not allow students to progress and develop mathematical thinking is the frustration they feel when they do not know basic mathematical facts or "get stuck" in calculations. When they cannot quickly perform simple additions and subtractions, they usually have additional difficulty in solving more complex problems that involve these operations. For this reason, it is important to systematically review terms, concepts, and procedures that have been previously taught.

Students can practice basic calculations at the beginning of each lesson (for approximately 10 minutes), mainly through playful activities (e.g., dice, dominoes, bingo).

For example, in a game with dice, children are asked to represent mathematical expressions of addition and subtraction, using concrete objects, visual representations, and mathematical symbols. These can be cubes, number lines, dominoes, grids, drawings or written mathematical expressions.

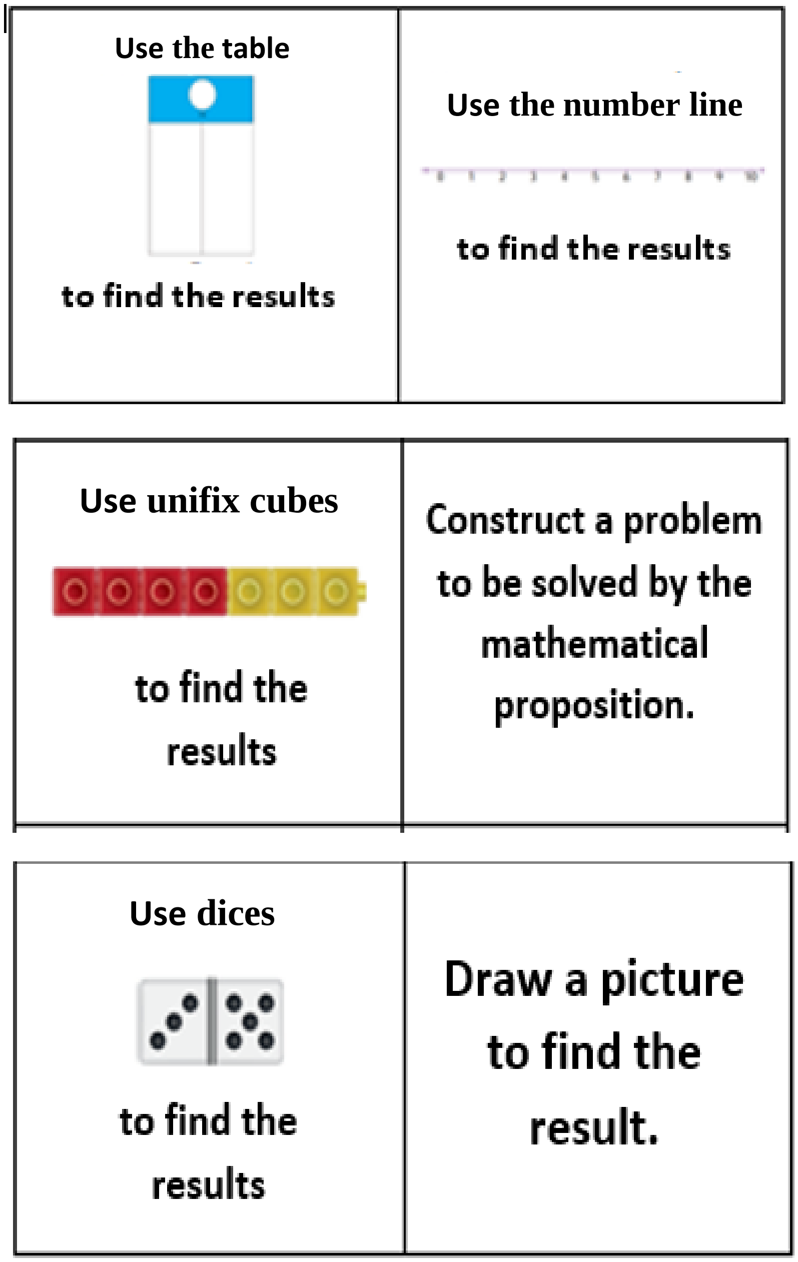


Figure 2.5: Different games for practice.

3. TEACHING ACTIVITIES

Introduction

In this section, we present activities that support students with learning difficulties to develop knowledge about numbers, operations with numbers, and patterns. Specifically, the topics we address include: (a) patterns, (b) number sense, (c) addition and subtraction meaning, (d) word problems, (e) number composition and decomposition, and (g) addition and subtraction strategies.

Each sub-section describes in detail how to teach mathematical concepts to students with difficulties in mathematics, by taking into consideration their difficulties and by finding approaches that meet their needs.

Furthermore, the teaching approaches discussed in Chapter 2 are taken into consideration (see Teaching methodology, p. 6), and examples of the mathematical tasks designed in the context of the research project are presented.

3.1 PATTERNS

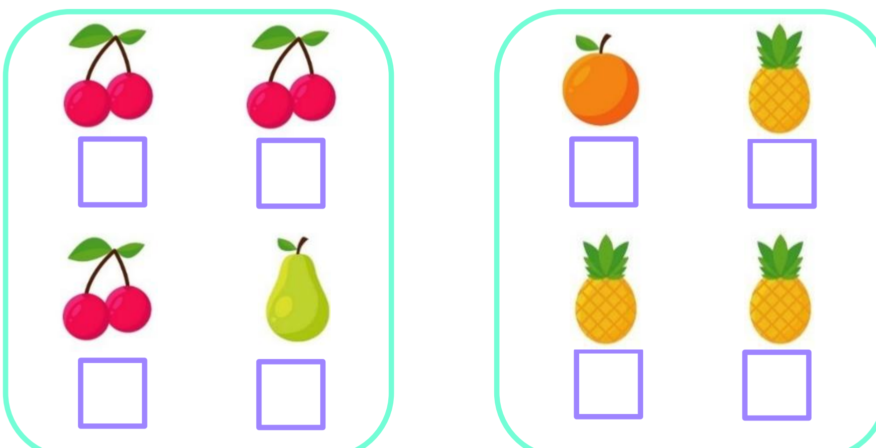
3.1.1 Introduction

Studying patterns is an important topic in mathematics since it helps students develop their algebraic thinking as well as their mathematical reasoning (Mulligan & Mitchelmore, 2009; Warren, 2005,). Pattern activities with young students are designed to turn their attention to identifying, reconstructing, and extending patterns (Van de Walle, 2007). The findings of numerous research studies provide evidence that young students are able to recognize, repeat, complete, extend, or construct patterns using a variety of materials (e.g., Papic et al., 2011; Rittle-Johnson et al, 2013; Skoumpourdi, 2013; Tzekaki & Kouleli, 2007). Moreover, students' difficulties in mathematics at later ages may occur due to the delay in engaging with patterns and looking for and detecting structural elements through them (Mulligan & Mitchelmore, 2009; Warren & Cooper, 2008).

3.1.2 Finding Similarities and Differences

The teaching of patterns in Grade 1 is carried out by introducing students to the recognition of similarities and differences between objects. First, we present students with small groups of similar objects that belong to the same category but differ with respect to a single criterion. For example, a group of fruits, cherries, and pears, is shown. Most of the fruits are cherries except from one fruit that differs, the pear (see Example 3.1).

Example: Mark (✓) the fruit that is different in each group.



Questions:

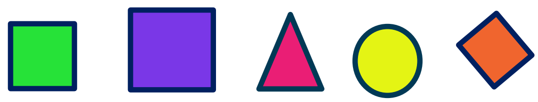
« Which fruit has different color? »
« Which fruit has different shape? »

Example 3.1: Activity to recognize differences between objects.

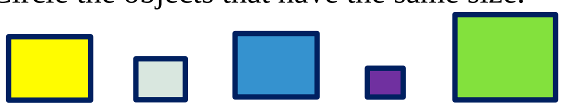
Then, students are asked to identify similarities between objects. The teacher presents small groups with different objects, where some of them have a common characteristic (e.g., same color, same shape, same size), as shown in the example below:

Circle the objects that have the same color.

Circle the objects that have the same shape.



Circle the objects that have the same size.



<u>Questions:</u>

« Which objects have the same color? »

« Which objects have the same shape? »

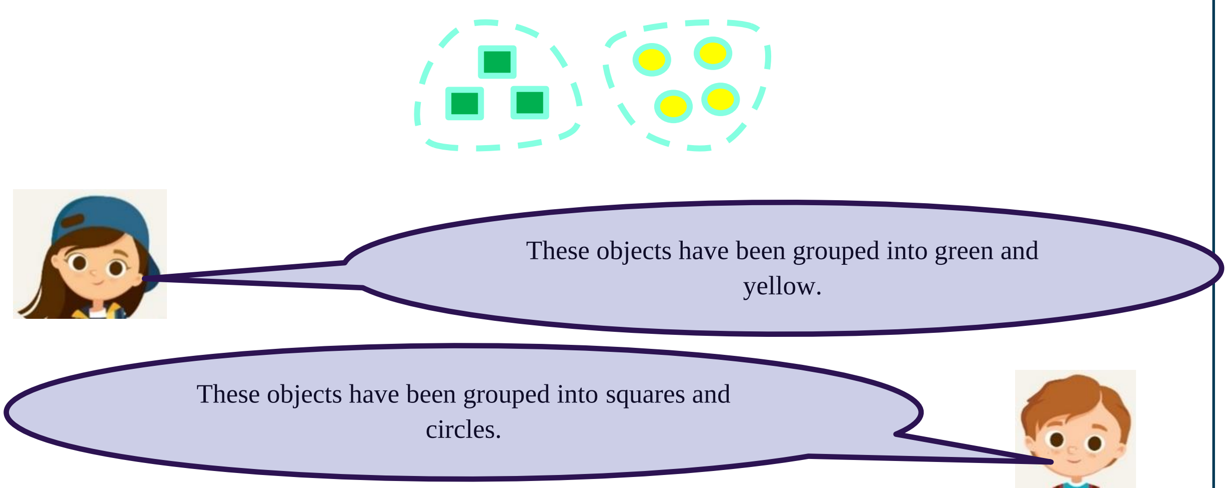
« Which objects have the same size? »

Example 3.2: Activity to recognize similarities between objects.

3.1.3 Grouping Objects

Students are gradually presented with small groups of objects that belong to the same category and can be grouped by color, shape, or size, as shown in example 3.3. The use of similar activities and scaffolding questions will support students in grouping objects by selecting appropriate criteria.

There are different ways to group objects!



Questions:

«How can these shapes be grouped? »

« Can they be grouped into green and yellow? »

« Can they be grouped into greens and yellows? …so, they can be grouped based on their color. »

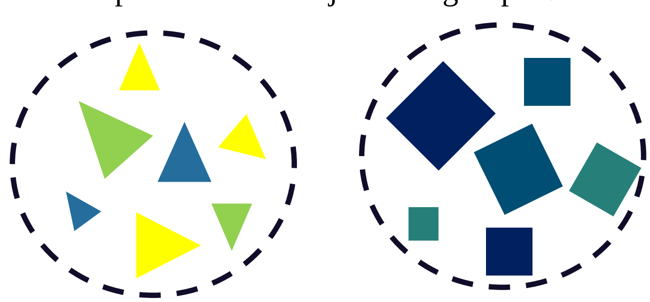
«Can they be grouped into circles and squares? … So, they can be grouped based on their shape. »

How can the objects be grouped?

Example 3.3: Grouping activity based on specific criteria.

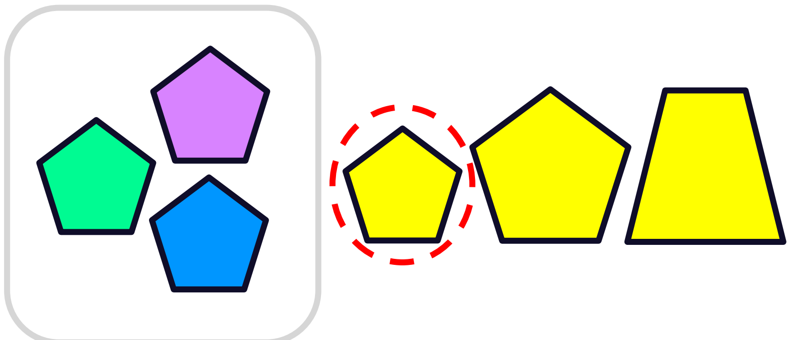
Students are presented with small groups of objects that have a common characteristic, such as color, shape, or size, and they are asked to identify the criteria by which the objects were grouped, as shown in the example below.

Explain how the objects are grouped.



Questions: « How were the objects grouped? »

Circle the shape that belongs to the group.



Questions:

«What do the shapes in this group have in common?

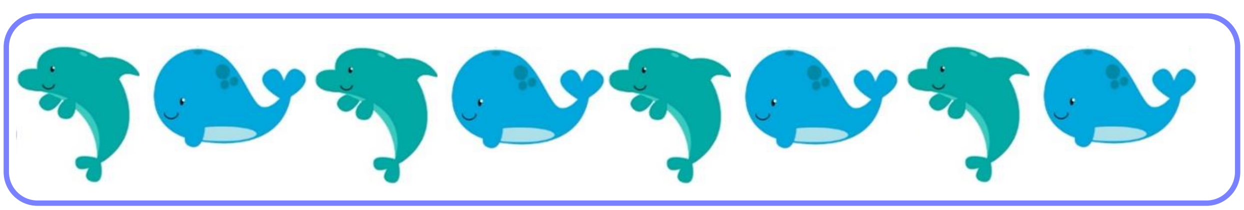
Which shape belongs in this group? »

Example 3.4: Shape Grouping Activity

3.1.4 Recognizing and Extending Patterns

Students in Grade 1 are expected to identify simple pictorial patterns. In the beginning, students are asked to identify patterns made of two repeating items (AB). As shown in the following example, a pattern recognition activity can start with a pattern of the form AB, continue with patterns of the form AAB, and then with patterns of the form ABB. In this way, students will work with patterns of increasing difficulty.

Describe each pattern.



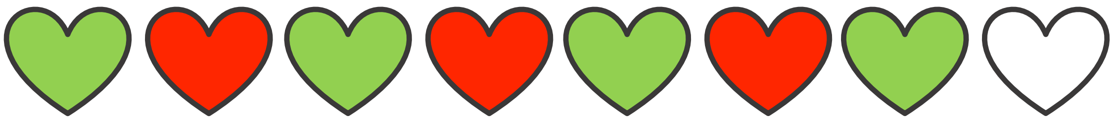
Questions:

« Here there is a dolphin (we show the first term), a whale, a dolphin...
can you continue the pattern? »

Example 3.5: Pattern recognition activity

In the next stage of teaching, students are given patterns of the form AB, in order to recognize the rule and extend them, as shown in example 3.6.

Look at the pattern carefully and color the last shape.



Questions:

« What do you notice in the pattern? »

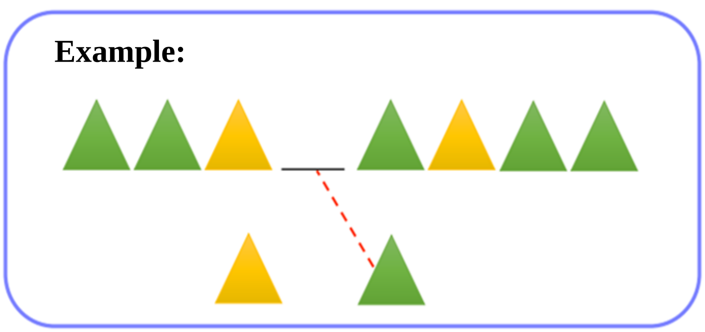
«What color will the heart be to continue the pattern? »

Example 3.6: Pattern Extension Activity

✓ 3.1.5 Completing the Patterns

As soon as students become able to recognize and extend simple patterns, other types of pattern's forms, i.e., AAB, BBA, ABB, BAA, ABC can be gradually introduced. During teaching, color patterns of all the above forms can be presented, for example yellow – yellow – blue, red – blue – blue, yellow – blue – red, and so on. The image below shows an activity where students are required to identify the rule based on a specific criterion (color). At the same time, they need to consider the place of the missing item within the sequence.

Fill in the pattern by selecting the correct triangle.



Questions:

« Which rule does the pattern follow? »

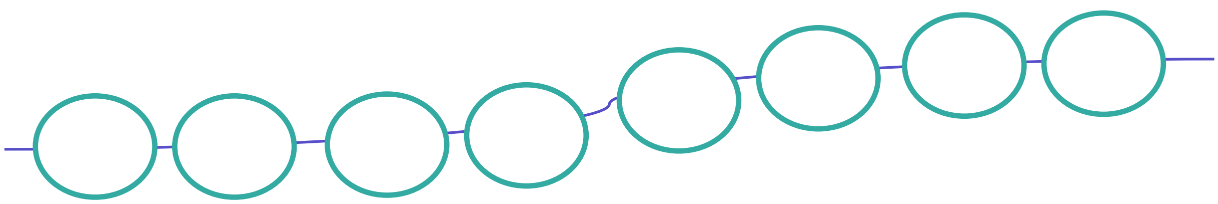
« What color should the triangle be so that the pattern continues? »

Example 3.7: Pattern rule finding activity.

3.1.6 Constructing Patterns

In the final stage, students are asked to create their own patterns. The teaching begins with given frames that students can modify, depending on the pattern they want to create. The teacher can ask students to create different patterns and observe the progress of each student individually. Below is an example of the given frames.

Color the shapes to create your own pattern.



Questions:

« How do you make the pattern?... You start with blue color, then red, red, blue...

Now which color should come next? »

« What is the rule of the pattern you have constructed? »

Example 3.8: Pattern making activity.

3.2 NUMBER SENSE

3.2.1 Introduction

Learning begins by presenting both objects and pictures for students. Starting with small numbers up to 5, we move to numbers up to 10, and represent them in different ways. In addition, emphasis is placed on writing numbers. Finally, teaching focuses on ordering and comparing numbers up to 10. In the following section, a proposed teaching sequence is presented. In particular, we present activities related to counting up to 10, connection of the verbal, quantitative, and symbolic forms of numbers, and automatic number recognition. Next, teaching number sense in mixed groups of objects, representing numbers up to 10, and writing numbers, are explained.

3.2.2 Counting up to 10.

We start by showing students small quantities (1 to 5 objects) and then larger ones (6 to 10 objects). We enumerate each group of objects out loud, so that students realize that there is a one-to-one correspondence. That is, as we count one by one, we point to each object. The objects within each group are identical, so that students remain focused on the enumeration of the group of objects.

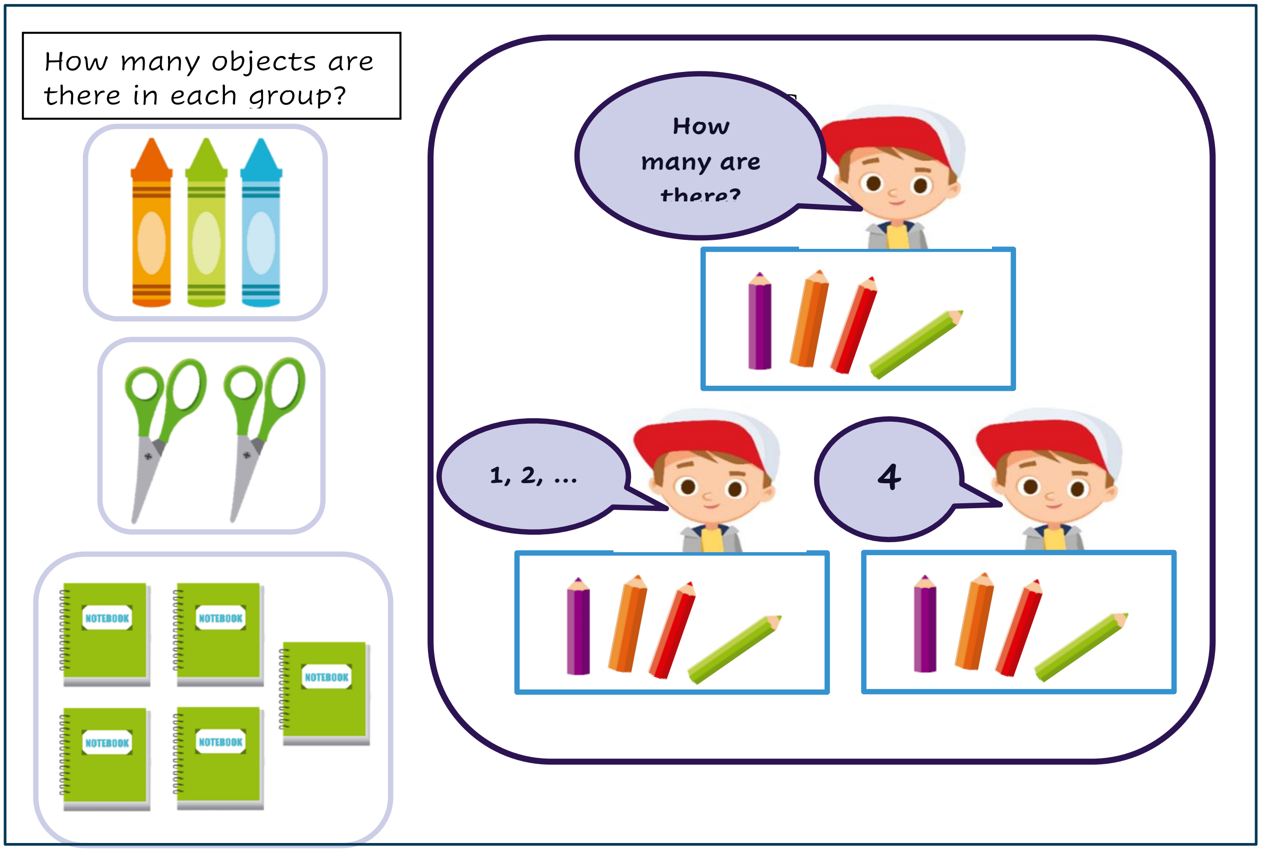
We constantly connect the enumeration process with the question: "How many?" After several examples, we invite students to count the number of objects in the same way. We encourage students to describe their thinking, using complete sentences, as in Examples 3.9 and 3.10.

“How are there?”

"One, two, three, four... there are four boxes."

"One, two, three, four, five... there are five peppers."

Example 3.9: Counting objects with one-to-one correspondence.

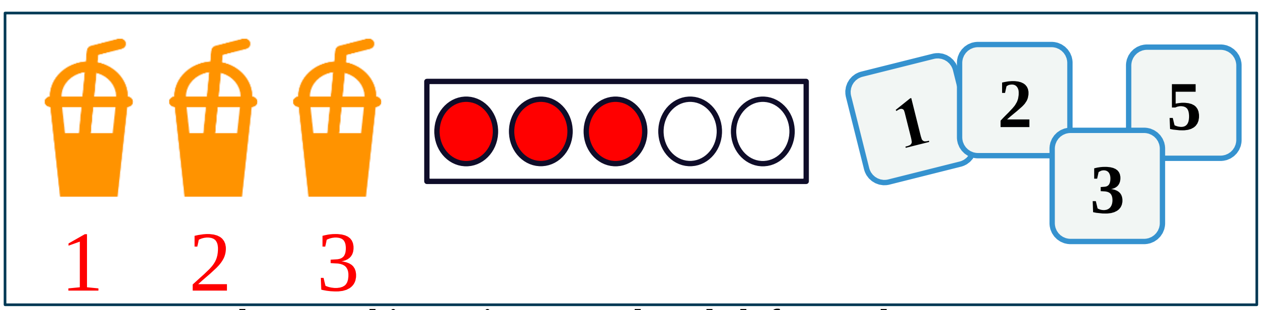


Example 3.10: Mathematical task for counting.

3.2.3 The verbal-quantitative-symbolic form of numbers

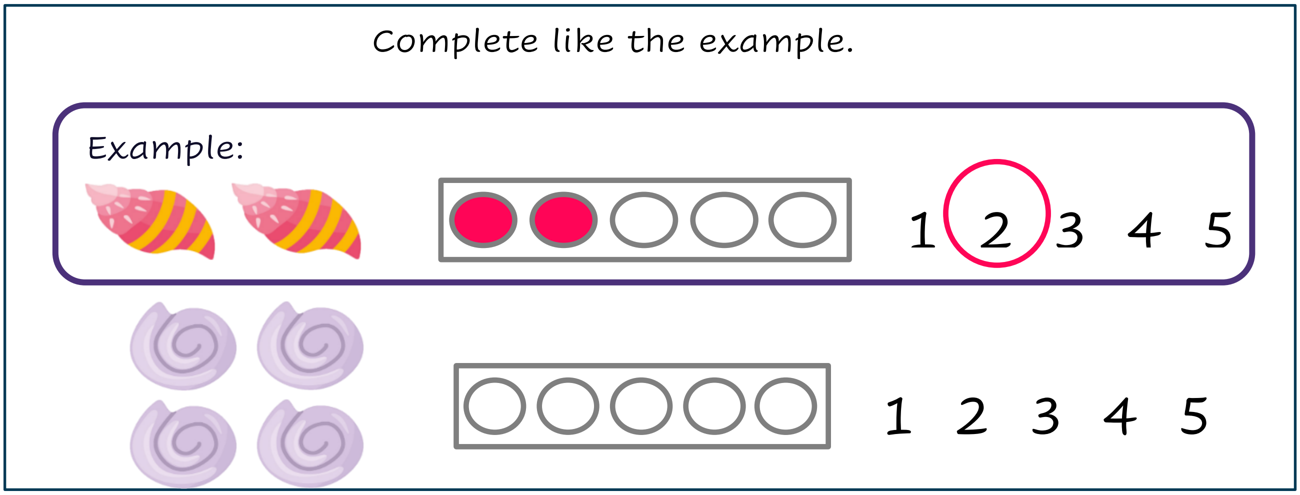
Teachers use different manipulatives to encourage making connections between the verbal, quantitative, and symbolic forms of numbers:

- Concrete, discrete objects
- 5-grid frames (for numbers up to 5) and 10-grid frames (for numbers up to 10).
- Cards presenting the symbolic form of numbers.



Example 3.11: Objects, pictures, and symbols for numbers up to ten

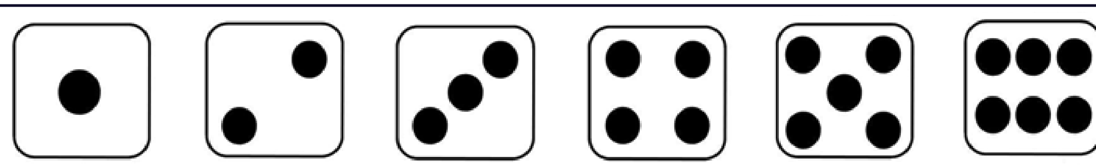
Teachers ask students to count the discrete objects, color on the grid as many dots as the number in the set and choose the corresponding number symbol.



Example 3.12: Mathematical task for number sense up to 5.

✓ 3.2.4 Subitizing

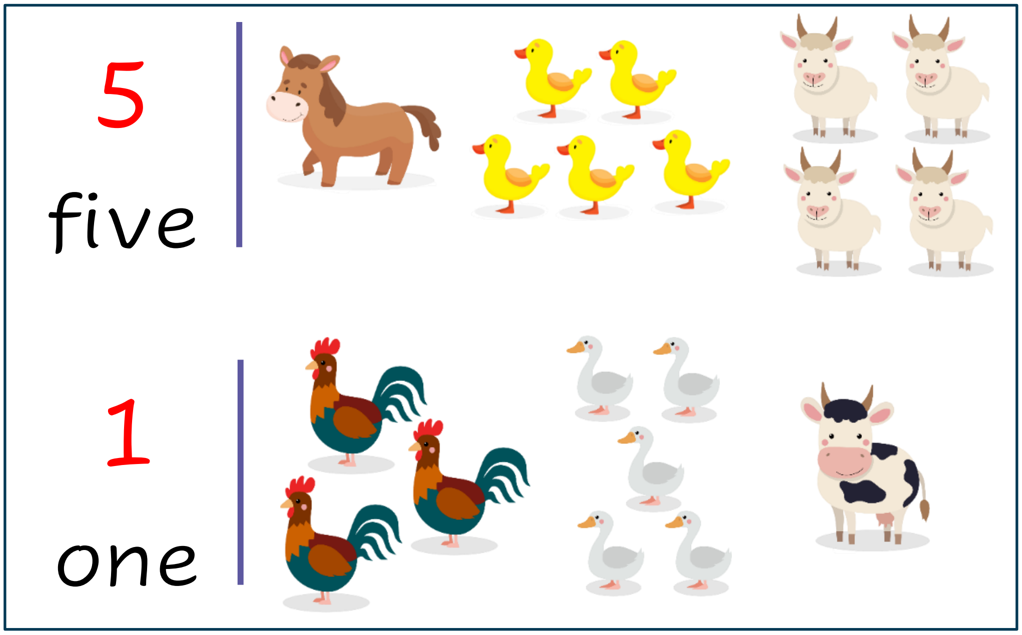
Teachers use pictures of discrete objects that are organized in a way that brings the number structure to the forefront. For this purpose, teachers can use objects, such as dice or domino, and pictures that present the structure of numbers to students in a familiar format (Example 3.13). To motivate students to immediately recognize the numbers without counting them one by one, we can ask supportive questions as shown below.



Questions:

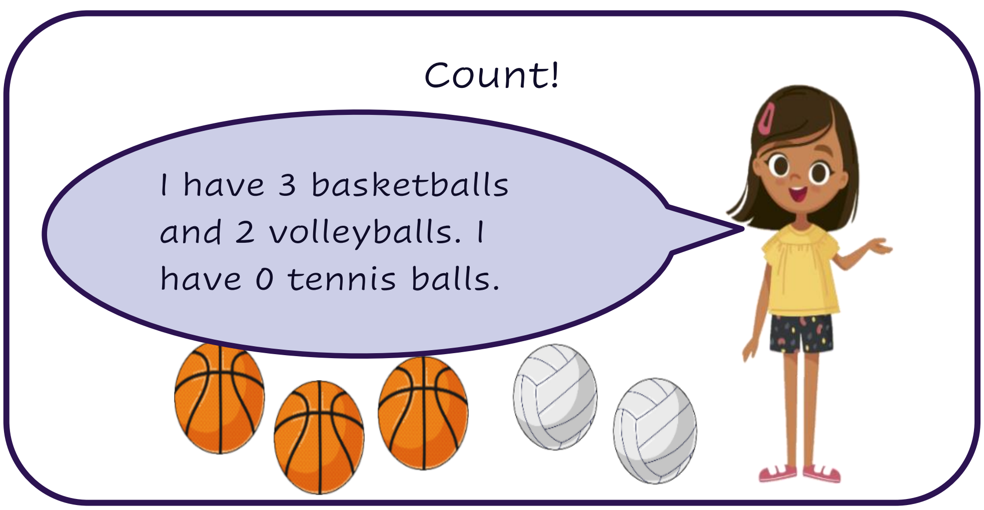
"Is it possible to find how many dots are shown without counting?"

Example 3.13: Number sense based on the structure of numbers through pictures.

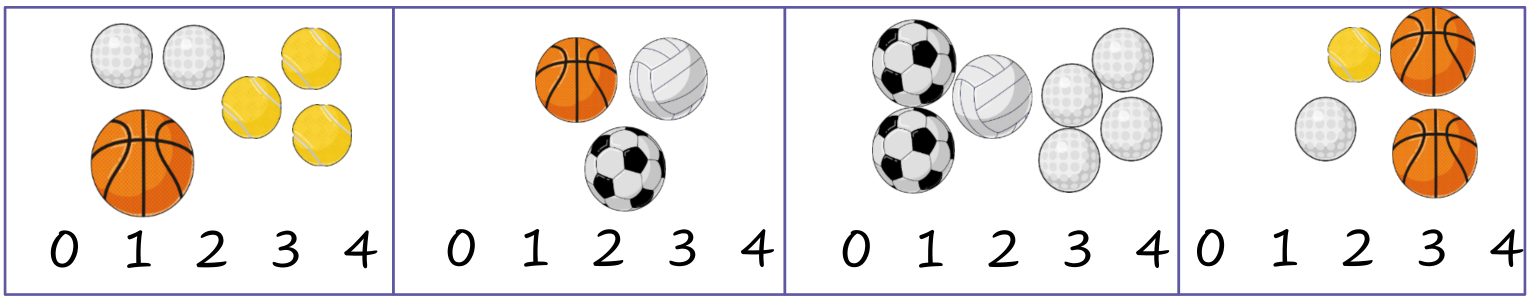


Example 3.14: Mathematical task for number sense

At a later stage, teachers present students with groups of mixed objects that belong in the same category. For example, different balls, flowers, and colored pencils. Students are asked to identify the quantity of each number represented in the picture. To help students identify the quantity of objects in different subgroups of a set, we can ask questions, as shown in Example 3.15.



Circle the number of the tennis balls.

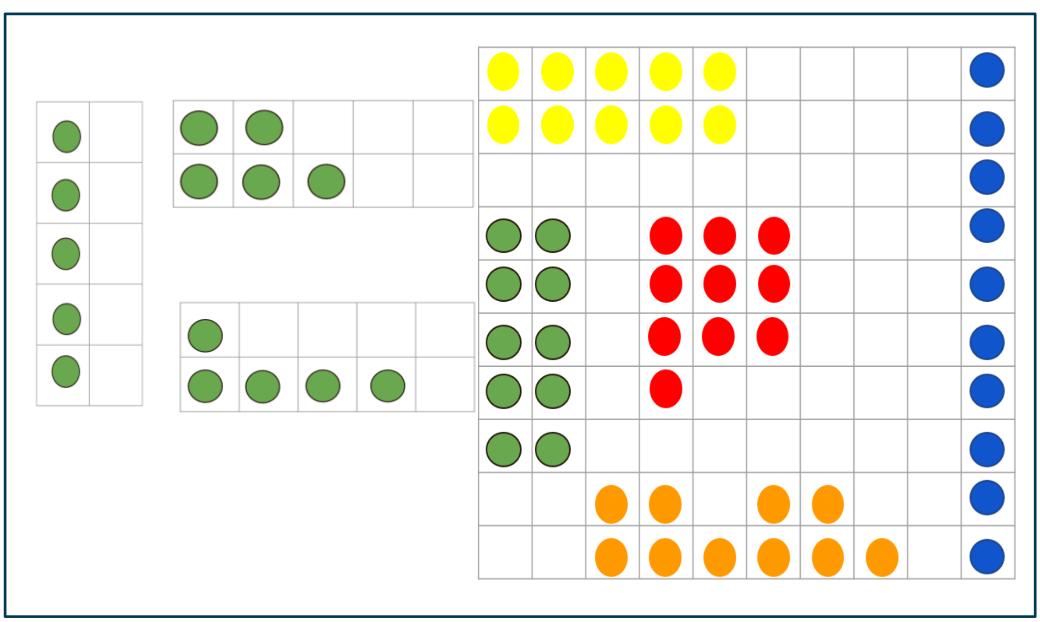


Questions:
"How many tennis balls are there?"
"How many basketballs are there?"
"How many volleyballs are there?"
"How many are all the balls?"

Example 3.15: Mixed groups of objects for number sense

✓ 3.2.5 Representation of numbers up to 10

Once students are able to list and recognize numbers up to 10, representation of numbers up to 10 is pursued, which helps students understand the multiple relationships of numbers. Students are presented with a 10x10 grid, on which they are encouraged to represent the numbers in various ways, as shown in Example 3.16. Students are encouraged to describe their thinking using comprehensive sentences, like "The number 5 can be formed by 2 and 2 and 1", "Number 5 can be formed by 1 and 4".



Questions:

"In what ways can you draw 5 dots?"

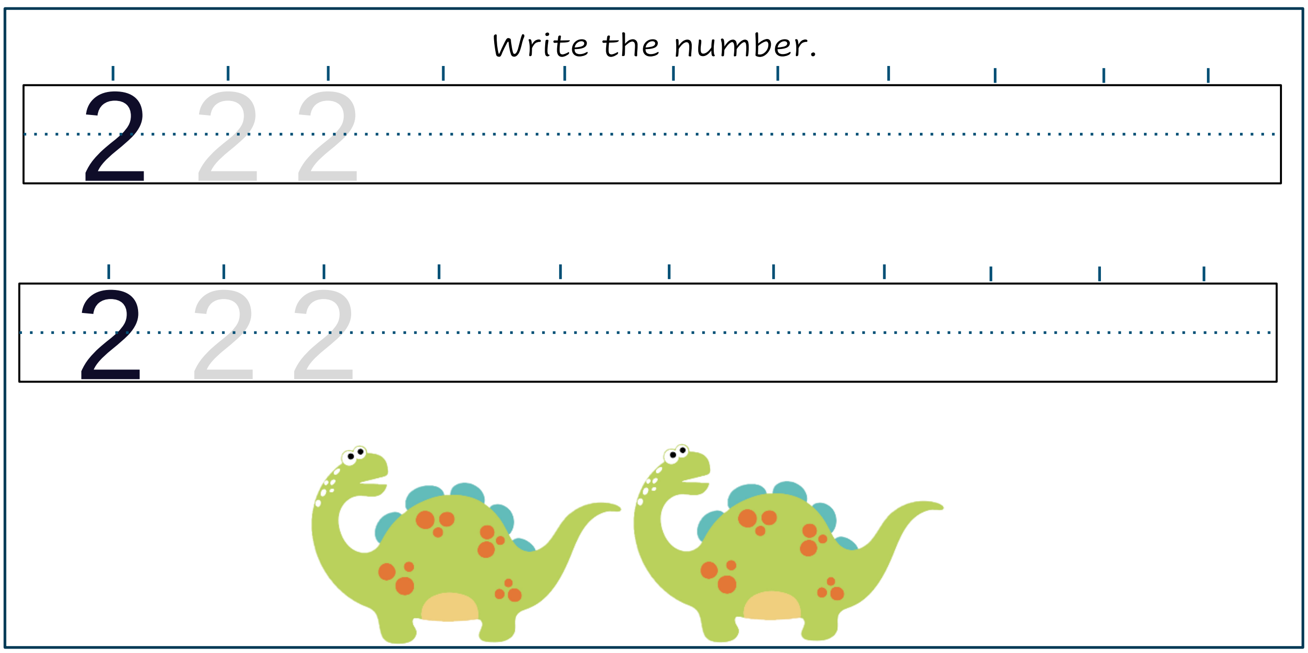
"In what ways can you draw 10 dots?"

Example 3.16: Representation numbers on grids

✓ 3.2.6 Writing numbers up to 10

Teachers emphasize the importance of writing the symbolic form of numbers correctly, while providing time for students to practice through structured writing exercises.

Teachers always ensure that connections are established between the symbolic form of the numbers and pictorial representations of the quantity it represents, in order to give meaning to the symbol.



Example 3.17: Mathematical task for writing numbers

3.3. SORTING—COMPARING NUMBERS.

3.3.1 Introduction

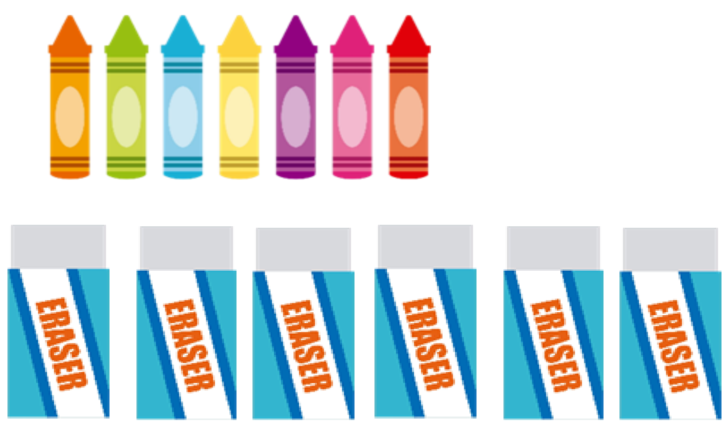
Teaching about sorting and comparing numbers focuses first on comparing two sets of objects at the concrete level and then using pictures.

Students are then asked to find the set with the largest or smaller number of objects, first by using clearly structured groups and then groups that are not arranged according to a specific structure.

Finally, students are asked to compare and order numbers as these are presented in symbolic form. Below, the teaching approaches for ordering and comparing numbers are described.

3.3.2 Equality

Teachers present to the students' objects that are placed in two sets, in such a way that they can compare them applying one-to-one correspondence. It is critical that all the objects belong to the same category, for example unifix of different colors. Students are encouraged to compare the two sets to see if they are equal or not, as shown below.



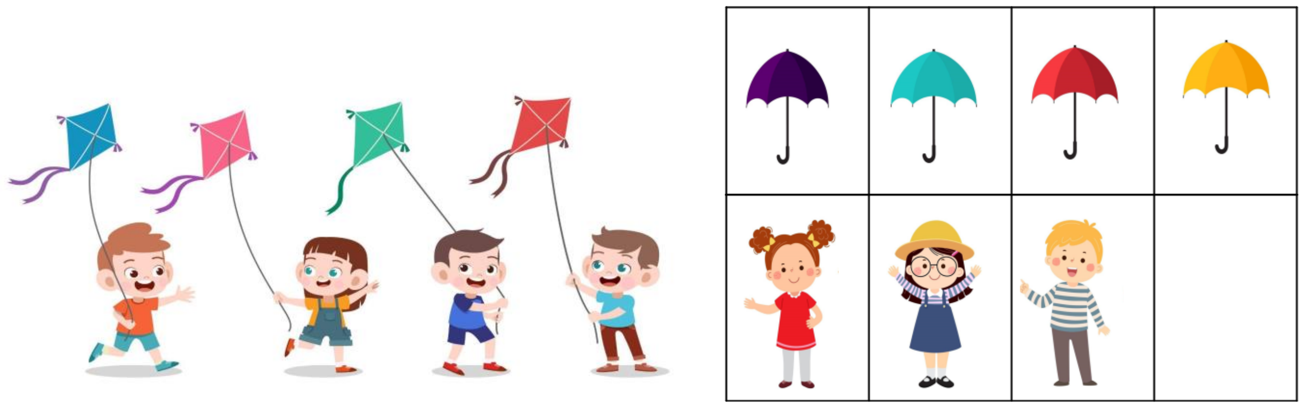
Questions:

"Are erasers as many as the pencils?"

"Is there a pencil for every eraser?"

Example 3.18: Comparison of two quantities at the concrete level

Teaching turns to the visual level using pictures. Teachers present pictures of two sets, in such a way so that students can match the objects in the two sets one by one to compare whether they are equal. Moreover, teachers can use a grid (see Example 3.19) to help students place the objects in a series, compare them with one-to-one correspondence, and check whether they are equal.



Questions:

"Are kites as many as children?"

"Is there a kite for every child?"

"Are umbrellas as many as children?"

"Is there an umbrella for every child?"

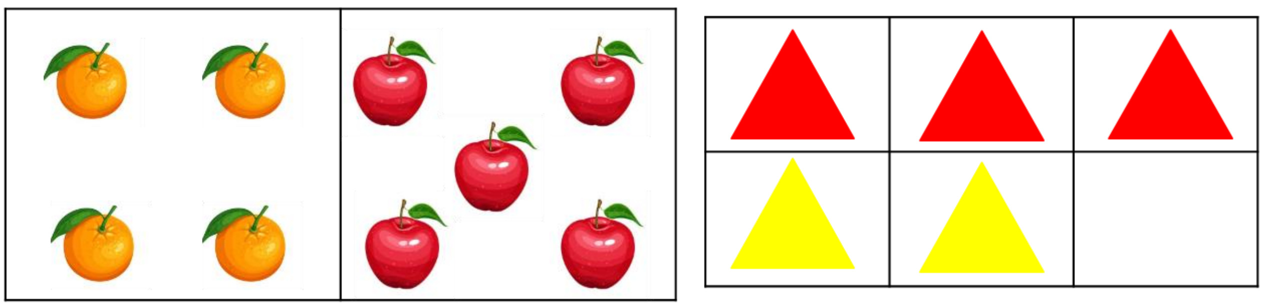
Example 3.19: Comparison of two quantities.

3.3.3 More-Less

Following activities on comparing numbers, teachers introduce activities in which students are asked to identify the set with the largest number of objects. We can present students with similar activities to the previous ones.

In this case, teachers encourage students to compare the two groups and find the group with the largest number of objects. Teaching is focused on the concrete and visual levels, using discrete groups of objects that belong to the same category but differ in color, size, or shape (see Example 3.20). Using a grid allows students to compare the two quantities and notice which group has more objects.

Finally, teachers encourage students to find the group with the smallest number of objects. Teachers may pose helpful questions to direct students' thinking towards comparing the groups of objects.



Questions:

"How many red triangles are there?"

"How many yellow triangles are there?"

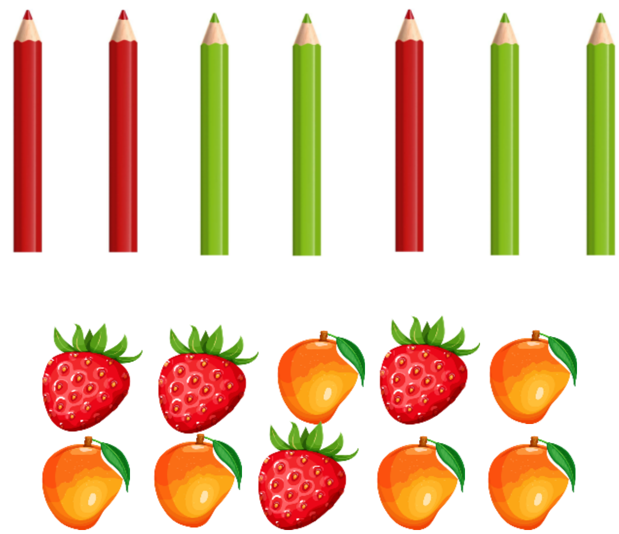
"Which are the most triangles, the red or the yellow?"

"Which are the fewest triangles, the red or the yellow?"

Example 3.20: Comparison of groups

At a later stage, teachers present students with mixed groups of objects to find which one has the largest or smaller number of objects. Teachers present the objects arranged as a mixed group, as shown in Example 3.21.

In this teaching phase, teachers include activities in which students are asked to identify whether the two sets are equal or whether group A has more or fewer objects than group B. To help students compare the mixed groups of objects, teachers can ask students questions as shown below.



Questions:

"How many red pencils are there?"

"How many green pencils are there?"

"Are red pencils more than green ones, or are they the same?"

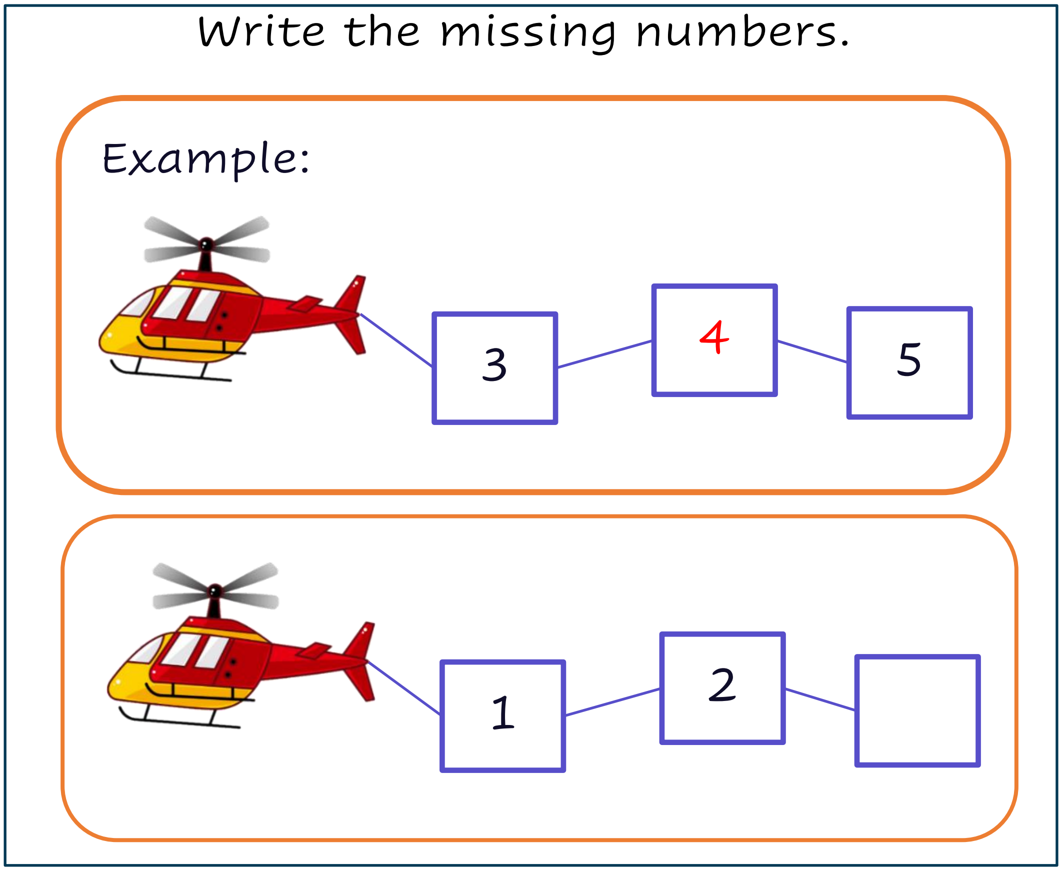
"Are strawberries less than mangoes, or are they the same?"

Example 3.21: Comparison of mixed groups of objects.

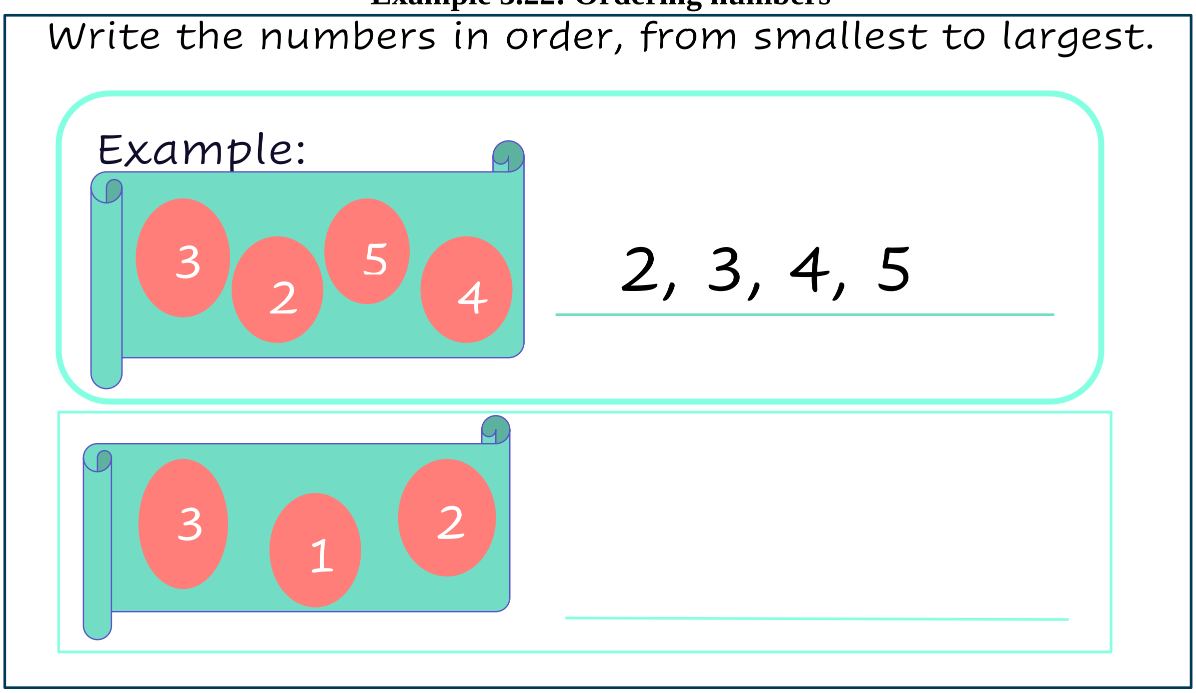
3.3.4 Writing numbers in order

After engaging students in comparing concrete objects and objects presented through images, teaching focuses on the symbolic level. Students are asked to sequence numbers up to ten, placing them in order from the smallest to largest.

First, teachers present to students three numbers, and they ask them to compare them and place them in order starting from the smallest one. Then, teachers present students with more than three numbers. In this kind of activities, teachers can also use number line tasks. Below, different examples of ordering numbers are presented (see Examples 3.22, 3.23 and 3.24).



Example 3.22: Ordering numbers



Example 3.23: Ordering numbers

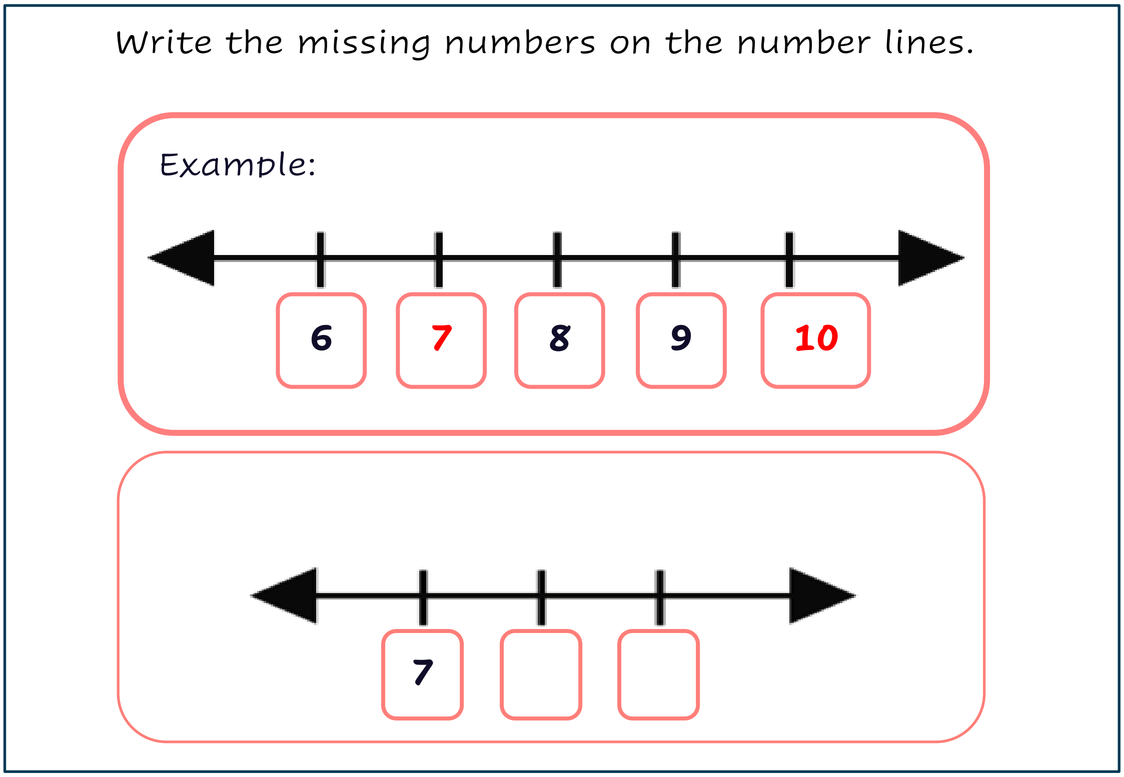


Figure 3.24: Ordering numbers using number line.

3.4 ADDITION AND SUBTRACTION

3.4.1 Introduction

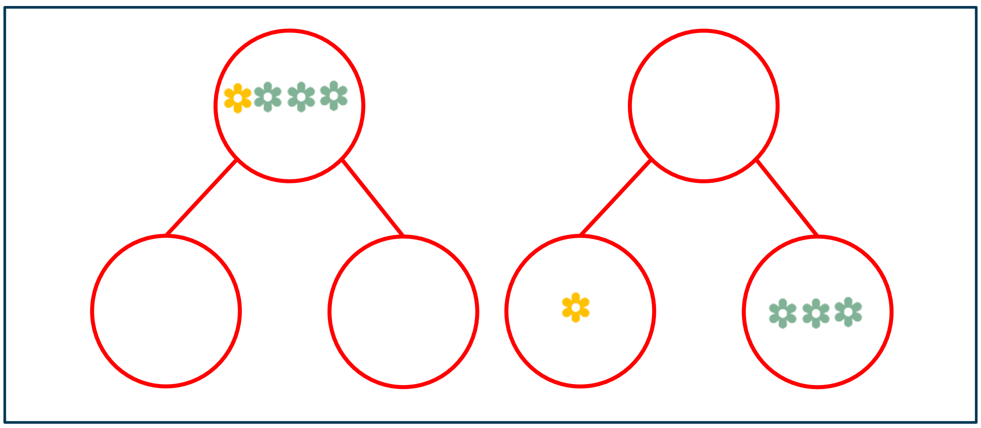
In this section, teaching approaches for addition and subtraction are presented. The teaching is initially focused on addition through the concrete, the pictorial, and the symbolic levels. Then, addition and subtraction stories are introduced. Finally, emphasis is placed on the relationship between addition and subtraction as opposite operations.

3.4.2 Addition

The teaching begins by introducing students to the whole, which can be divided into two parts and vice versa. Teaching then focuses on students' understanding of addition as an equality relation between the whole and the two parts being added. Finally, teaching addresses the commutative property of addition. Specific teaching approaches to addition are described below.

Part-Whole. Teachers use objects and part-part-whole diagrams to introduce students to the concept of addition. Teachers organize the objects in such a way that the two addends are presented separately in two groups and students can move them together to create a whole. As shown in Example 3.25 teachers encourage students to write the two addends in symbolic form.

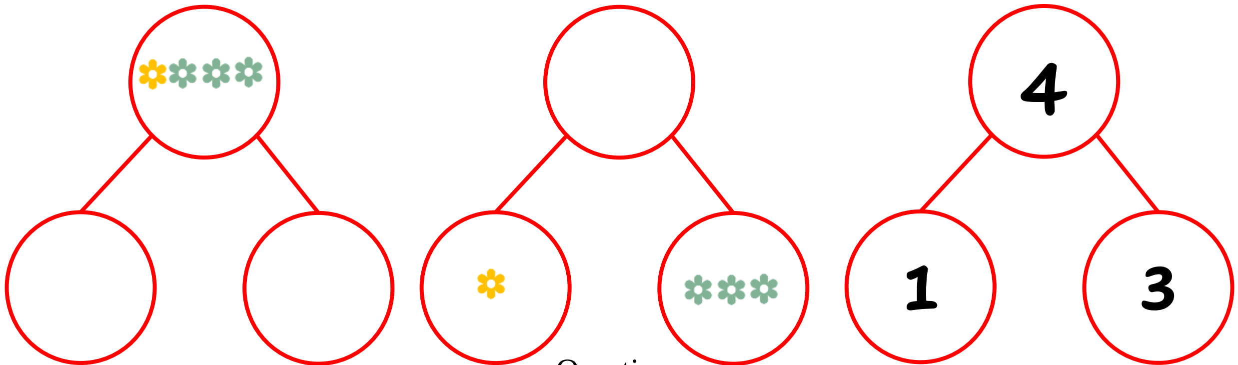
Students decompose the number by moving objects from the "whole" to the "parts" and they compose the number by moving objects from the "parts" to the "whole." The relationship between the "whole" and the "parts" represents the same quantity.



Example 3.25: Composition and decomposition of number 4, using objects and diagrams

At the same time, teachers use the same approach to show the process of using mathematical symbols. The emphasis is constantly placed on making connections between objects, pictures, and symbols.

Finally, students are encouraged to find as many ways as possible to decompose a number. At this teaching phase, it is important to use cubes of the same color and size so that students can turn their attention to breaking the number of cubes into subgroups, regardless of their color and size. Students are encouraged to describe the process of composing and decomposing numbers, as shown in Example 3.25. We also help students form complete sentences (Example 3.26).

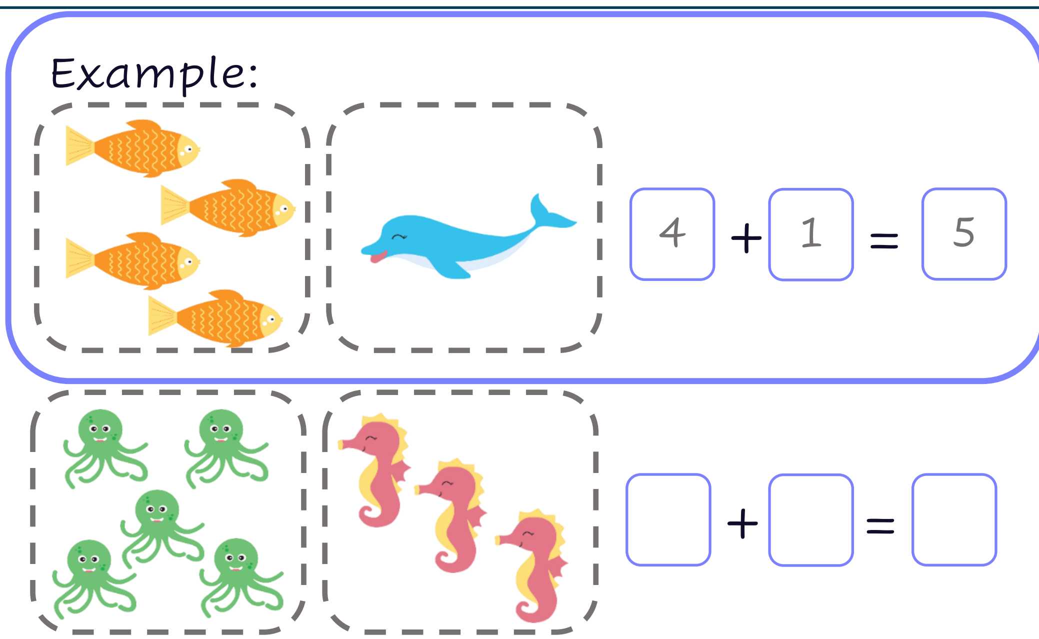


Questions:

"What does the number 4 represent?" "4 indicates the number of flowers."
"What does the number 1 represent?" "1 indicates the number of yellow flowers."
"What does the number 3 represent?" "3 indicates the number of purple flowers."

Example 3.26: Composition and decomposition of number 4, using materials and frames.

Equality of parts - whole. Pictures are presented to students that clearly show two sets to be added. Students are encouraged to write appropriate addition sentences based on pictures.



Questions:

"How many are the goldfish?"

"How many are the dolphins?"

"How many are all the fish?"

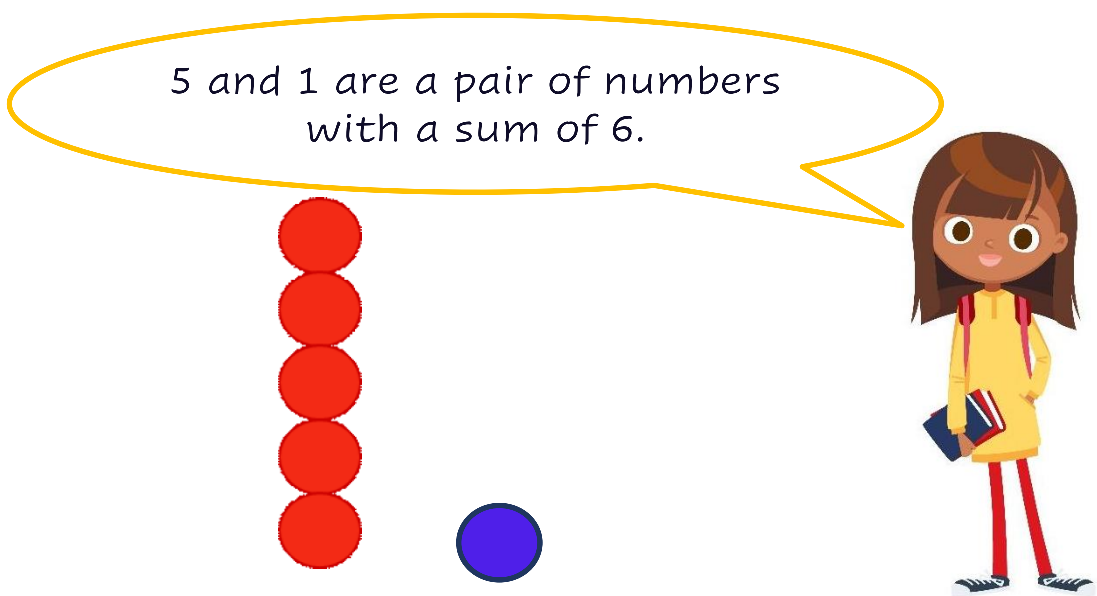
Example 3.27: Visual representation for addition. Equality of the "whole" and the two "parts".

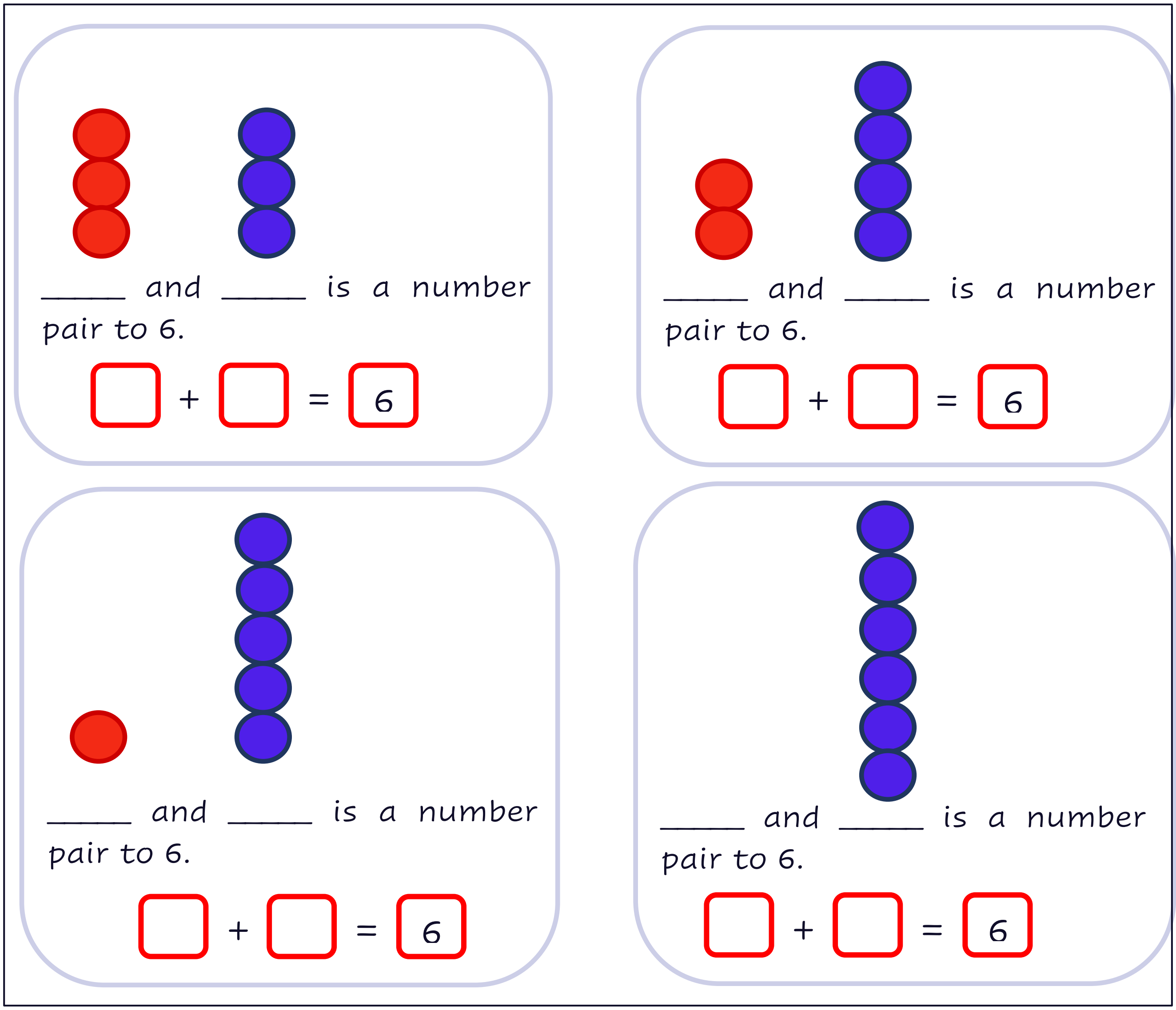
When students are asked to write mathematical sentences, we introduce the equals sign “= “and we name it, to show that the whole is equal to the sum of the two parts. Thus, teachers refer to the whole and note that it is possible for the whole to be divided into two parts. Teachers need to state that "Number 5 equals 2 and 3."

Pairs of numbers. A useful strategy for students to practice is finding the sum of pairs of “double” numbers, such as 2 + 2 = 4. Students with math difficulties may also have difficulty remembering the procedures for performing calculations. Therefore, having to remember two numbers (e.g., 2 and 4) instead of 3 (e.g., 2, 3, 5) is easier.

Students are also encouraged to find the pairs of numbers for a given sum, as shown in Example 3.41. Initially, students practice the pairs of numbers from 1 to 5. Later, the pairs of numbers from 6 to 10 are introduced.

Example:

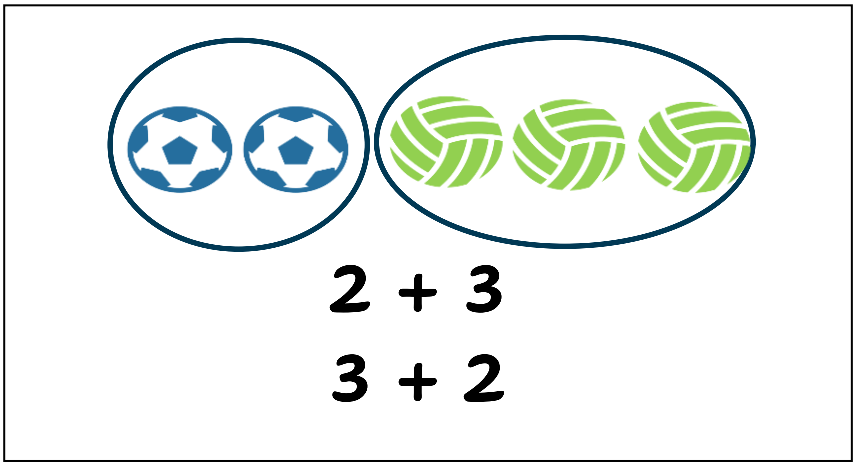




Example 3.28: Tasks on composition and decomposition of numbers to 6.

The commutative property. Teaching the commutative property through multiple examples allows students to see that the sum of two numbers is the same regardless of the order in which the numbers are placed. Thus, the students' memory is supported, as they have fewer sums to remember.

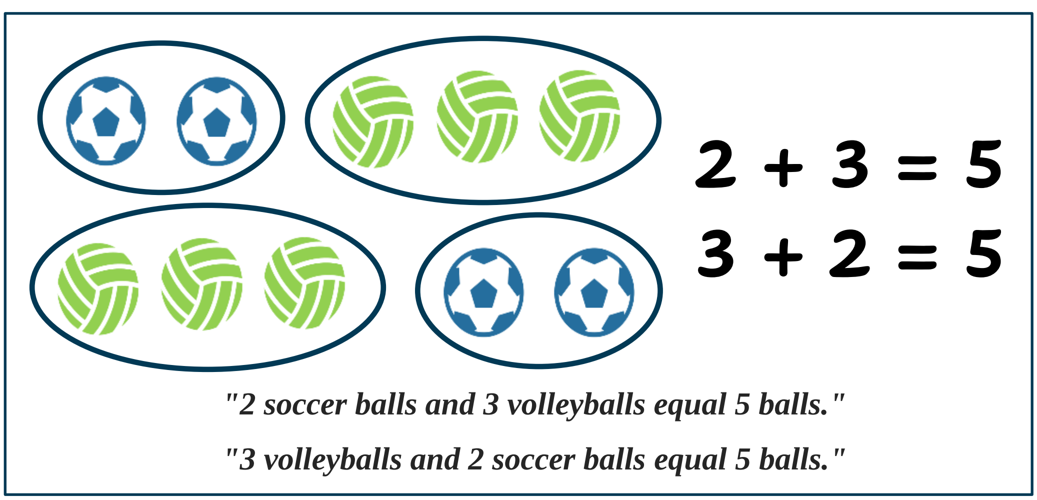
Teachers use objects which are presented in two groups. Students are encouraged to write the two addends in a symbolic form, as shown in Example 3.29.



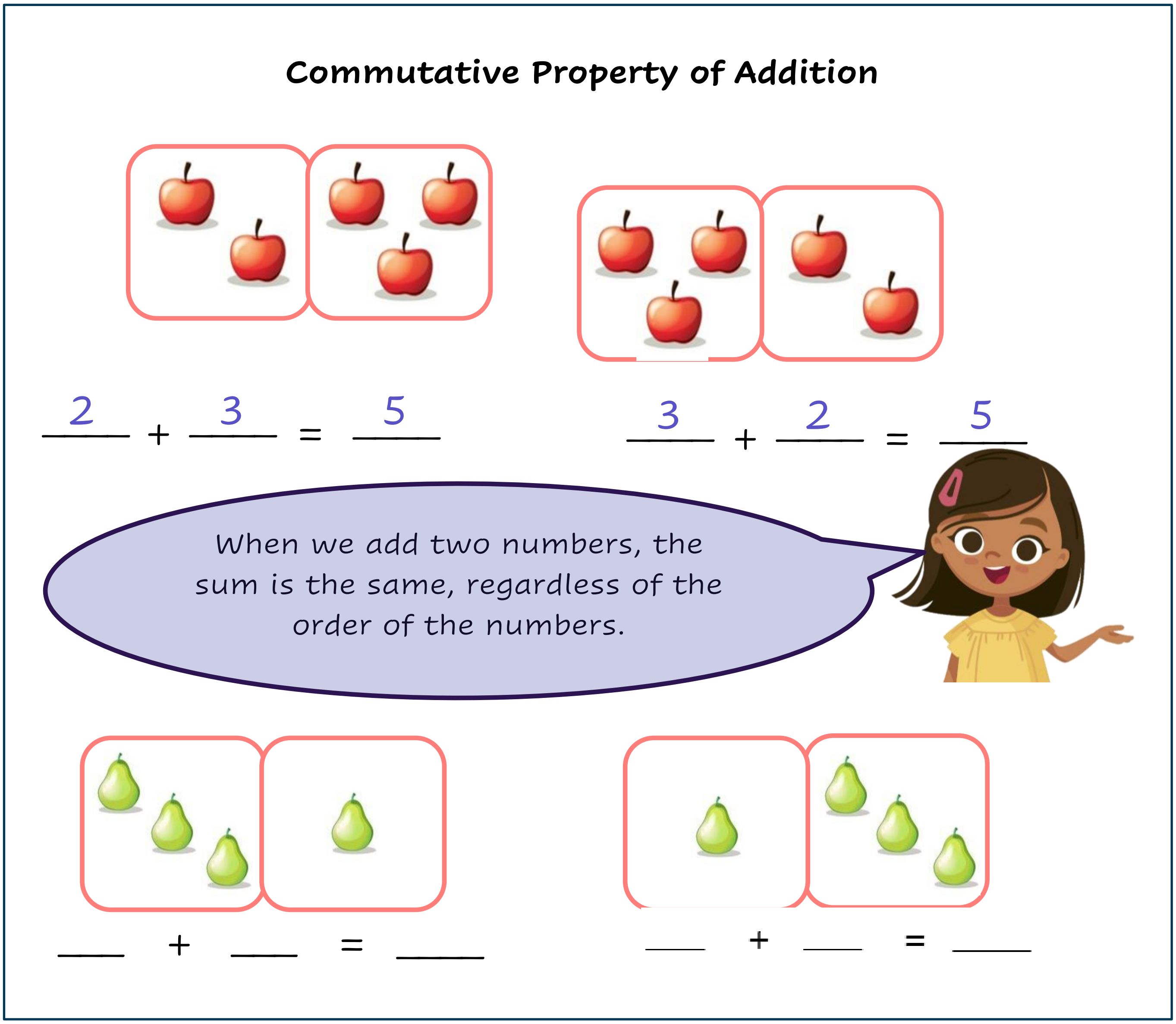
Example 3.29: Introductory task to addition using objects.

Teachers introduce students to the addition symbol, which they pronounce correctly as "plus." At the same time, teachers emphasize the word "add" in the verbal descriptions, such as *"There are 2 soccer balls and 3 volleyballs", "We write it as 2 plus 3, which means, I add 2 and 3", "2 represents the 2 soccer balls, and 3 represents the 3 volleyballs."*

Teachers use manipulatives (e.g., unifix) and encourage students to represent the two mathematical sentences, as shown in Example 3.30, in order to realize that the sum remains the same. Students are encouraged to write mathematical sentences to connect the visual representations of objects or images with the symbolic form. Next, teachers use activities with visual representations, as Example 3.31 shows.



Example 3.30: Commutative property using objects.



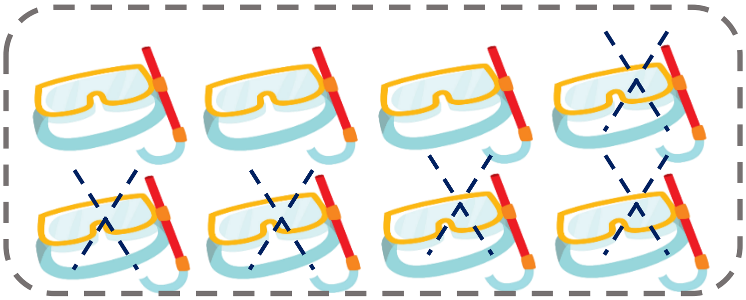
Example 3.31: Addition using the commutative property.

3.4.3 Subtraction

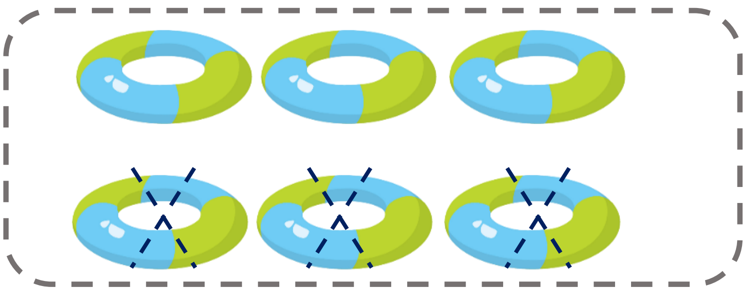
Introducing students to subtraction starts with the use of pictorial representations. Students are required to write subtraction sentences based on pictures that clearly demonstrate the minuend and how it was reduced. The students should find the set of number objects that were initially in the group, the set of objects that were deleted and the set of objects that remained (see Example 3.32).

Write the missing numbers in each mathematical sentence.

Example:



$\boxed{8} - \boxed{5} = \boxed{3}$



$\boxed{} - \boxed{} = \boxed{}$

<u>Questions:</u>

First: “How many lifejackets were there at the beginning?”

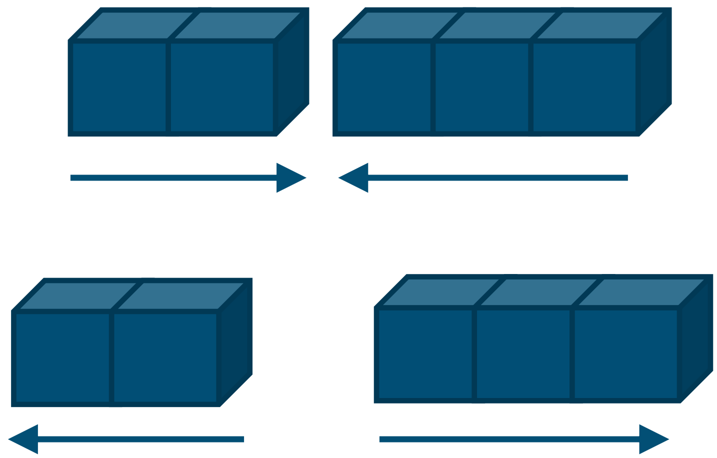
Later “How many lifejackets were later lost?”

Now: “How many lifejackets are left now?”

Example 3.32: Task for the introduction of subtraction

3.4.4 Addition- Subtraction as opposite operations

Unifix cubes can also be used when teaching addition and subtraction as opposite operations. Students are provided with a set of unifix cubes that have been divided in two smaller groups, as seen in Example 3.34. Teachers demonstrate to the students how to join the two pieces while emphasizing the term “addition,” and how to separate them while emphasizing the term “subtraction.”



Questions:

"I have 2 blue pencils. I bought 3 more pencils. How many pencils do I have now?"

"I had 5 pencils. I gave 2 pencils to my friend. How many pencils do I have left?"

Example 3.33: Aggregating and partitioning cubes, for addition and subtraction

Every time teachers give an addition-subtraction story to students and they write mathematical sentences. For example:

"I had 2 blue pencils. I bought 3 more pencils. How many pencils do I have now?"

$$2 + 3 = 5$$

Teachers also introduce the symbol "−" naming it "minus,", while using the word "subtract". For example:

"I had 5 pencils. I gave 2 pencils to my friend. How many pencils do I have left?"

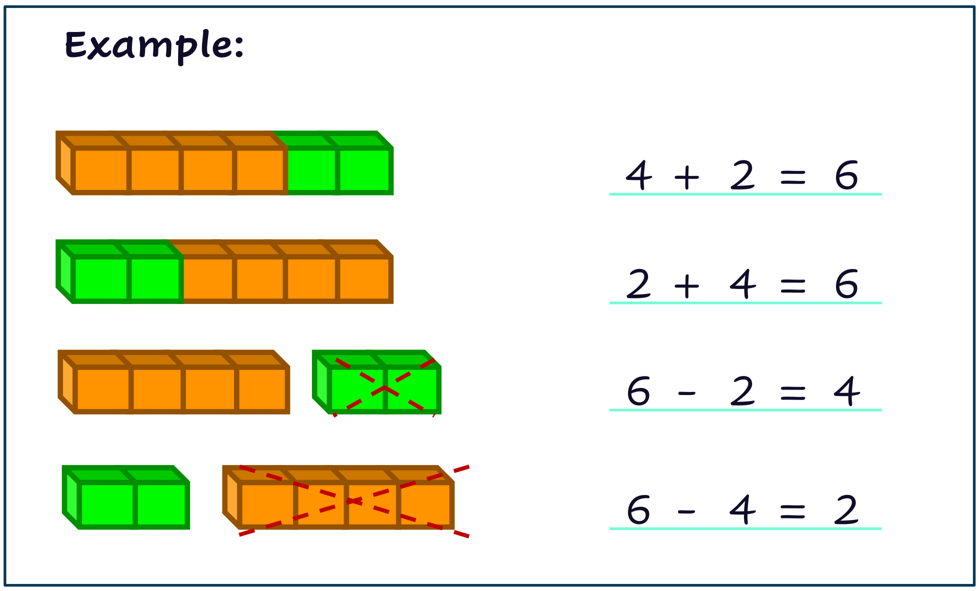
$$5 - 2 = 3$$

"Out of 5 pencils, we subtract 2. There are 3 pencils left."

5 minus 2 equals 3

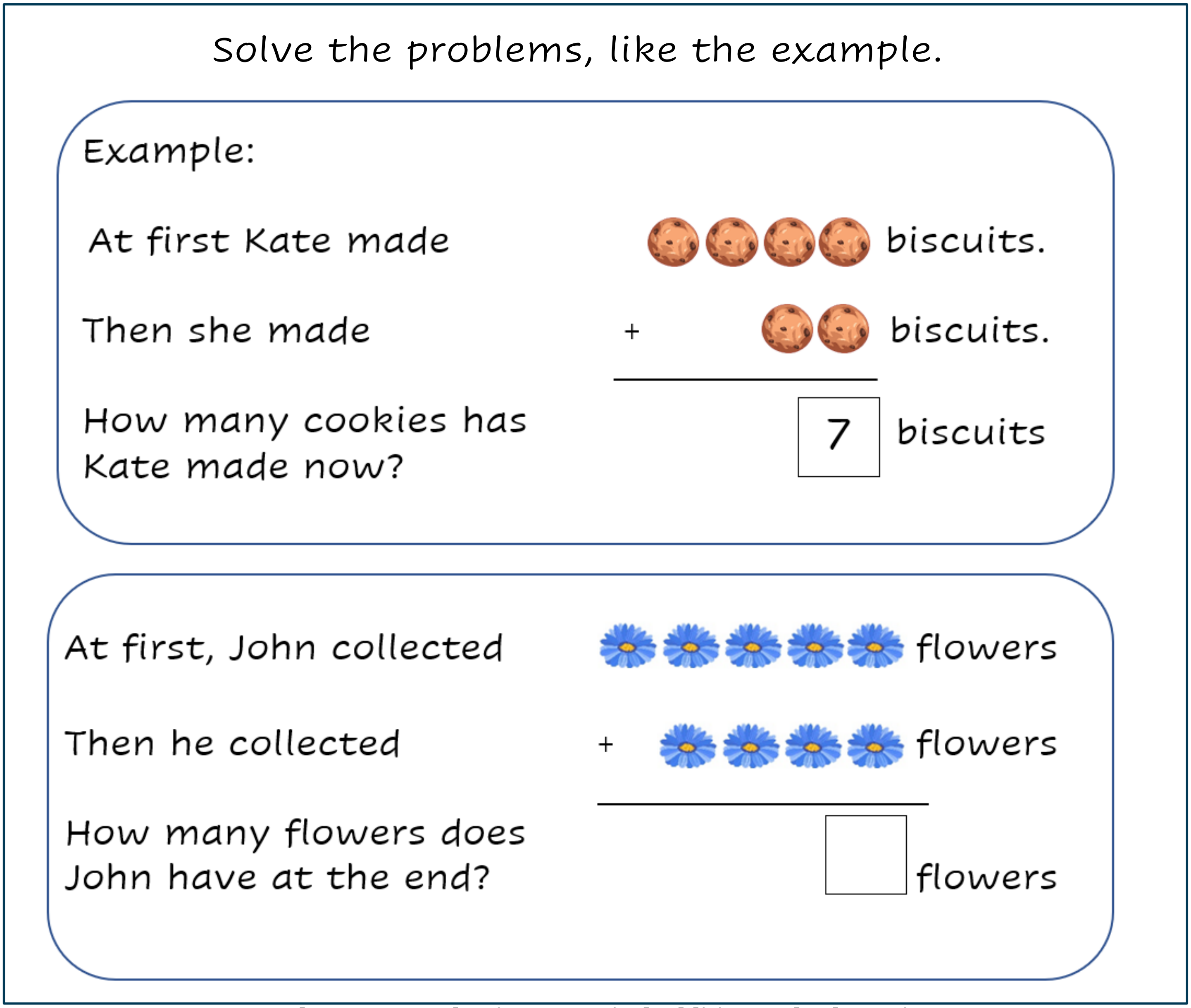
Fact families. The teaching of subtraction is presented through its relationship with addition and not as a separate domain. This helps students solve subtraction math problems without having to memorize any new knowledge.

Teachers encourage students to create a family of numbers (see Example 3.34). This allows students to recognize the relationship between numbers in addition and subtraction through the analysis and composition of numbers.



Example 3.34: Example for fact families.

Introduction to vertical addition and subtraction. One way to present vertical addition and subtraction in math problems is shown in Example 3.35. Teachers give students the opportunity to visualize the problem mathematically and to calculate the sum or difference vertically.



Example 3.35: Introduction to vertical addition and subtraction

3.5 ADDITION STRATEGIES

✓ *3.5.1 Introduction*

To help children that encounter memorization difficulties, we emphasize conceptual understanding and learning calculation strategies rather than memorizing sums and differences.

Teachers should aim to present students with simple ways to perform calculations without expecting that students with math difficulties will be able to discover many different strategies.

Below, two addition strategies are presented, counting on and adding using pairs of numbers.

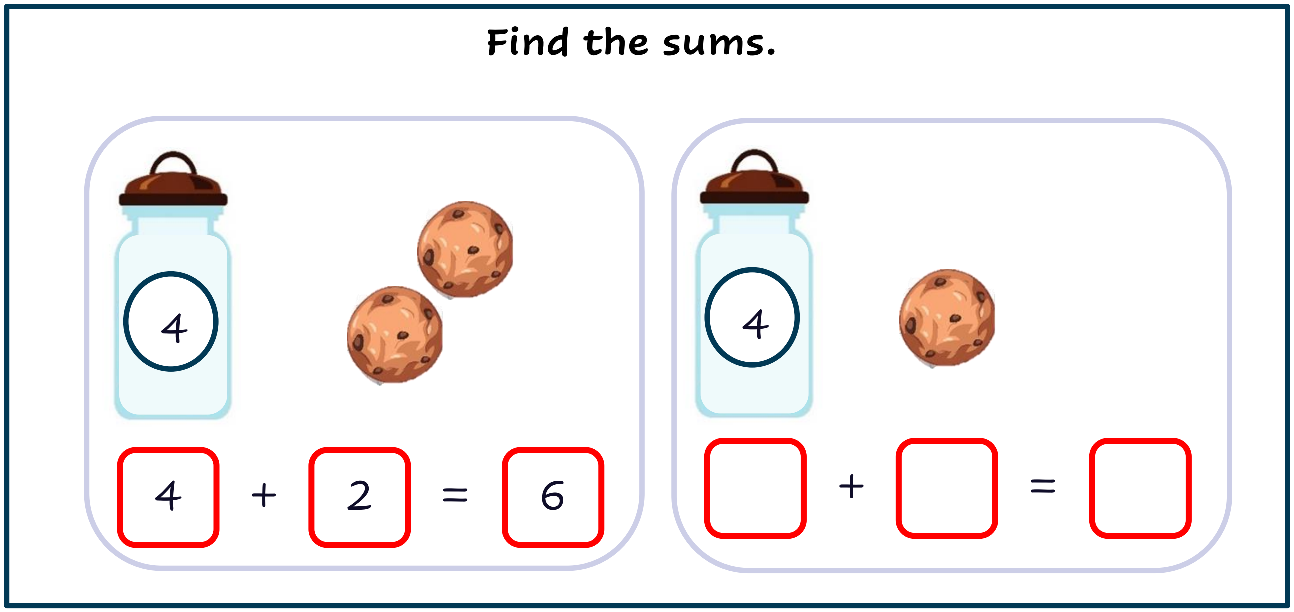
3.5.2 Counting up

Teaching this strategy will help students start counting from the biggest number. It is a strategy with which students with difficulties will be able to learn in order to find the sum more easily.

We can introduce the strategy with unifix cubes or other manipulatives (e.g., pencils, erasers) to help students to understand it.

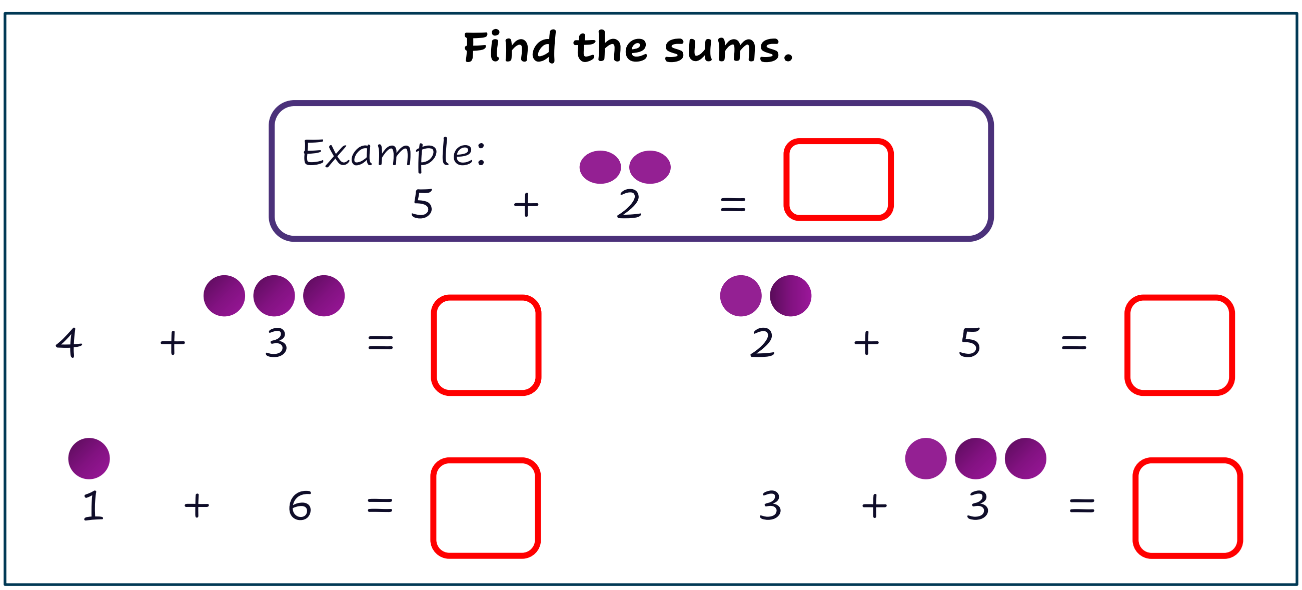
First, teachers present a closed and opaque container and stick a number on the outer surface. Teachers also place objects outside the closed box. Starting with the number written on the container, we count up according to the number of objects outside the container to find the total (see Example 3.36).

It is important to have a smaller number of items outside the box to encourage students to count up from the largest addend.



Example 3.36: Adding objects with the counting up strategy.

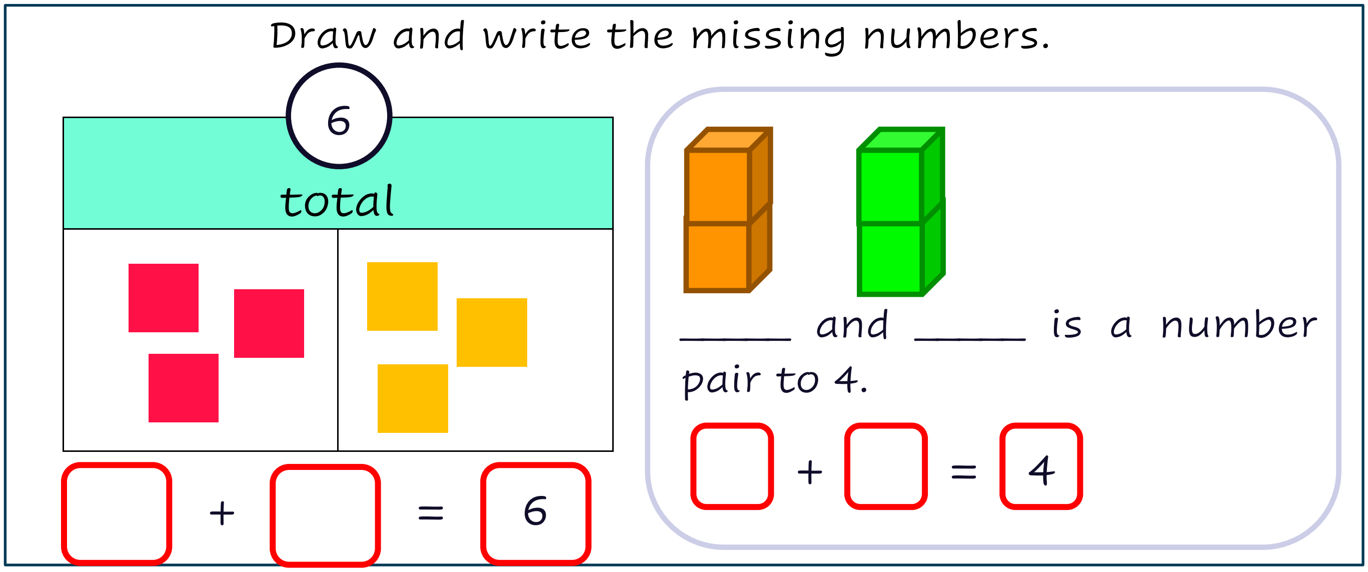
Teachers present to students various examples, moving from the concrete to the pictorial, and then to the symbolic level. Teachers present students with mathematical addition sentences, with the smallest quantitative addend being represented, either through the use of pictures or dots (see Example 3.37). Teachers encourage students to start with the first addend and count up based on the pictures to find the sum.



Example 3.37: Task for addition with counting up strategy.

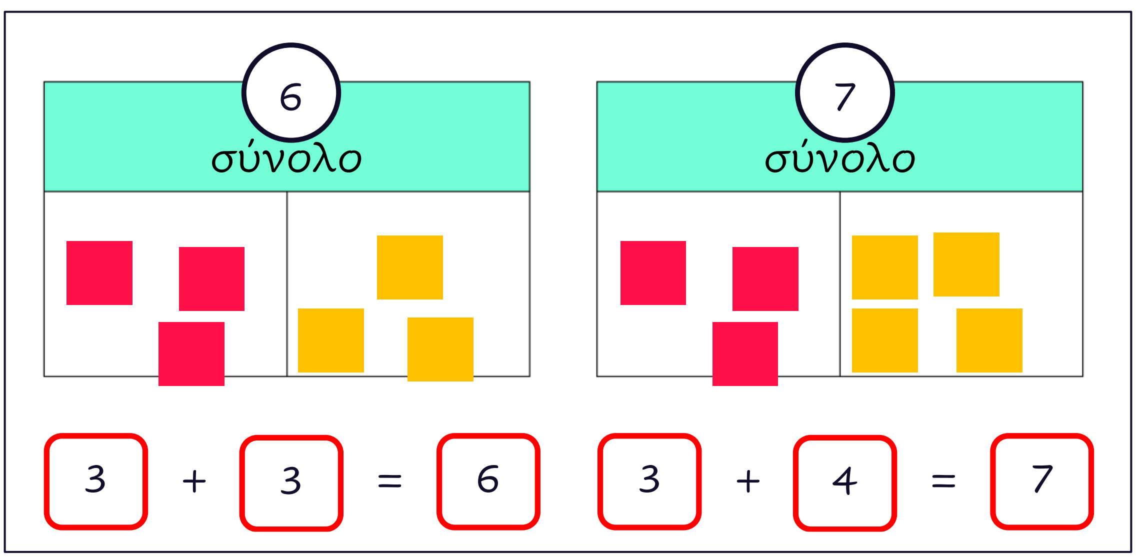
3.5.3 Double Numbers

An easy addition strategy is for students to practice is finding the sum when the addends are the same , such as 2 + 2 = 4. Students are encouraged to find the pairs of numbers so that a sum is an even number, as shown in Example 3.38.



Example 3.38: Task for addition with counting up strategy.

Later students work on addition tasks, as those shown in Example 3.39. To calculate 3+4, students are encouraged to first calculate the sum of 3+3 and then add one more unit.



Example 3.39: Task for addition with double numbers.

3.6 MATHEMATICAL WORD PROBLEMS

3.6.1 Introduction

Teaching problem solving begins with simple addition math stories involving either grouping or changing. Grouping and changing problems reflect either addition or subtraction, depending on the history of the problem. Below, the way of teaching the mathematical stories of addition and subtraction is analyzed, as well as the characteristics that mathematical problems need to incorporate to be understood by the students.

3.6.2. Problem characteristics

Teachers present mathematical problems to students, that describe addition and subtraction stories, to develop their problem-solving skills. The mathematical problems should have certain characteristics, so that they meet the needs of students with difficulties.

One-Step Problems:

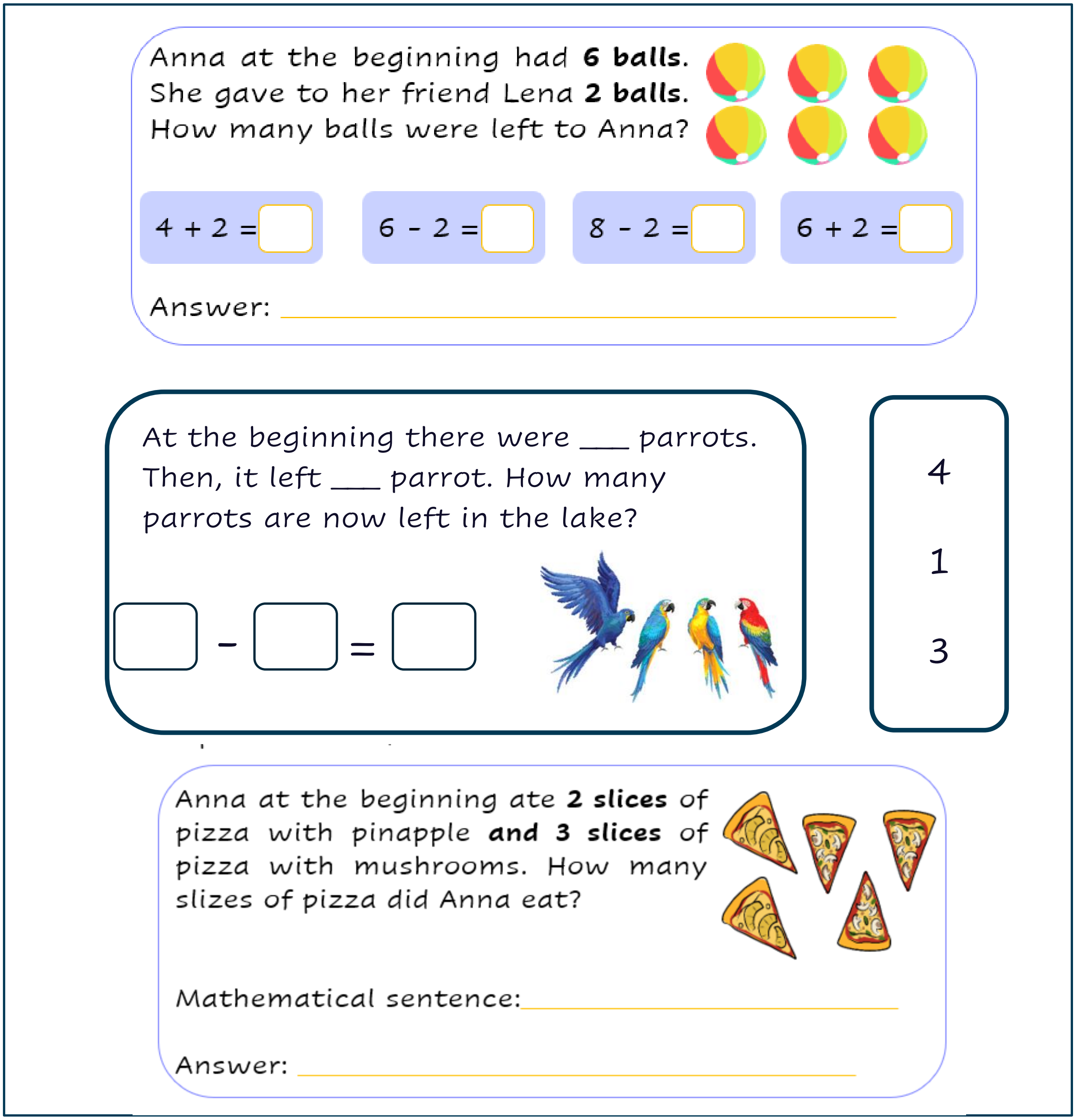
Considering students' difficulties with their working memory, we use problems that are solved with one mathematical sentence (one-step problems), for example, 7+2=. In this way, students are asked to solve a single mathematical proposition to find the solution, which will not confuse them.

Visual representations:

We use visual representations that are directly related to the problem description. Thus, we help students solve the problem by connecting the verbal description of the problem with pictures, which is easier for them to understand.

Simple vocabulary:

The problems should have a clear and short verbal description, with simple vocabulary, so that students can easily understand the problem to solve it, without getting confused by the description.



Example 3.40: Mathematical problem

3.6.3 Addition Problems

Teaching problem solving begins with the introduction of mathematical stories. Teachers first present grouping and change problems. Grouping problems involve two different sets of objects which will be added.

Change problems include an initial situation, the change that takes place, and the final situation. Change problems can be linked either to addition or subtraction (see Subsection: Subtraction stories), depending on whether the change is related to partitioning or jointing.

Grouping. We introduce students to simple stories, which can be gradually translated into mathematical addition sentences (see Example 3.41). Through the visual representations, the students are asked to answer short questions regarding the quantities involved in the mathematical proposition.

$\boxed{1} + \boxed{4} = \boxed{5}$

<u>Questions:</u>

"How many animals are outside the river?"

"How many animals are in the river?"

"How much are all the animals?"

Example 3.41: Grouping story for addition.

How many books Dora holds? $\boxed{}$

How many books Peter holds? $\boxed{}$

How many are all the books? $\boxed{}$

$\boxed{} + \boxed{} = \boxed{}$

Example 3.42: Mathematical addition story.

Change problems. For change addition stories, we emphasize the words "at the beginning", "then", "now" (see Example 3.43). We use these words to emphasize the change that a situation is facing.

At the beginning

Then

Now

$$\boxed{1} + \boxed{2} = \boxed{3}$$

<u>Questioning:</u>

First: ***"How many children were on the bus at the beginning?"***

Then: ***"How many children entered the bus next?"***

Now: ***"How many are all children on the bus now?"***

Example 3.43: Grouping story for addition.

3.6.4 Subtraction Problems

Mathematical stories of subtraction are mainly concerned with the change in a situation. Similarly, as in the mathematical change addition stories, we emphasize to students the words "at the beginning," "later," and "now". Gradually, students are encouraged to write subtraction math sentences, recognizing the quantity presented in the picture each time and formulating the mathematical sentence (see Example 3.44 and Example 3.45).

At the beginning

Then

$\boxed{6}$ $\boxed{4}$ $=$ $\boxed{2}$

Now

<u>Questions:</u>

First: ***"How many apples were on the apple tree at the beginning?***

Later: ***"How many apples did John cut later?"***

Now: ***How many apples are left now?"***

Example 3.44: Mathematical change subtraction story

First

Then

Now

How many flamingos were there in the lake at the beginning?

How many flamingos are not in the lake?

How many flamingos are now left in the lake?

$\square - \square = \square$

Example 3.45: Change story for subtraction.