Supporting 1st grade students with difficulties in Mathematics

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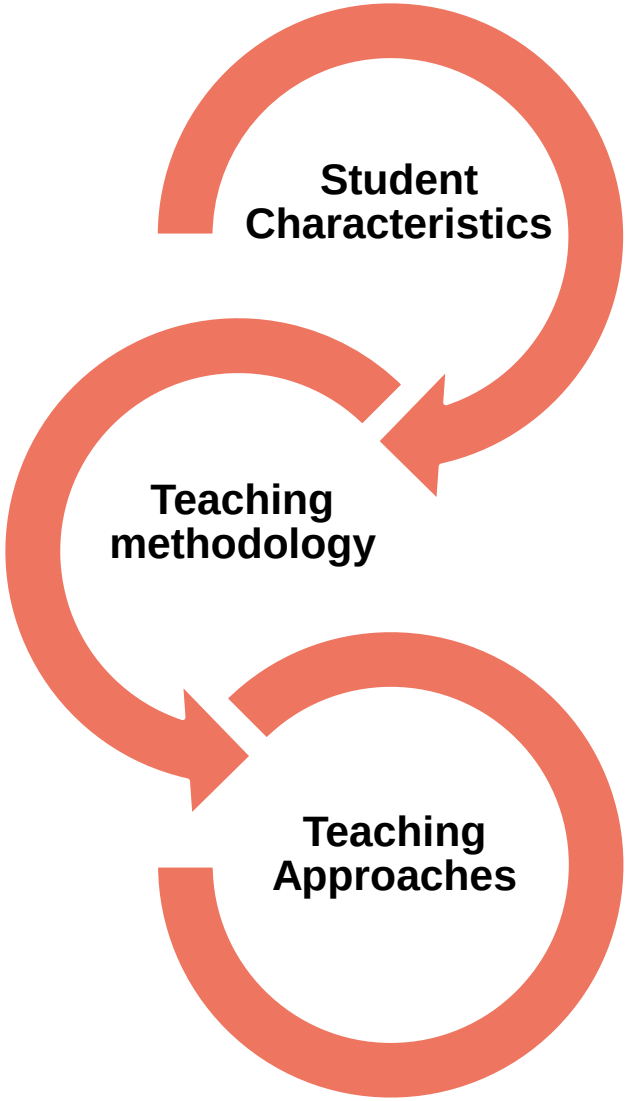
Remedial teaching courses for 1st grade students with difficulties in mathematics



Student Characteristics

1st grade students difficulties: Examples

- Slow speed at enumerating quantities
- Difficulty in enumerating quantities based on groupings (e.g., 2-2)
- Slow speed at writing numbers
- Difficulties in writing numbers correctly
- Difficulties in recognizing the symbolic form that represents a given quantity
- Lacking fluency in recalling the outcome of additions and subtractions up to 10 (e.g., 5+3, 10-8).

1st grade students difficulties: Examples

- Difficulties in reproducing procedures or rules
- Slow speed and/or mistakes when performing simple calculations due to reliance on counting quantities (e.g., the find the sum 5+3 by counting all 1,2,3,4,5…6,7,8)
- Difficulties in remembering mathematical vocabulary (e.g., the words ‘addition, subtraction, sum, difference’)

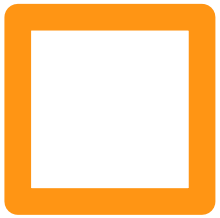
1st grade students difficulties: Examples

- Difficulties in understanding simple word problems (e.g. they miss a step of the process, distract from the goal)
- Difficulties in interpreting and constructing representations
- Difficulties in implementing procedures that involve multiple steps.

Teaching methodology

Teaching methodology

- Three levels of representation
- Clear-cut learning objectives
- Examples from everyday life
- Multiple examples
- Questioning
- Mathematical tools and technology
- Mathematical terminology
- Systematic repetition
- Reflection
- Assessment



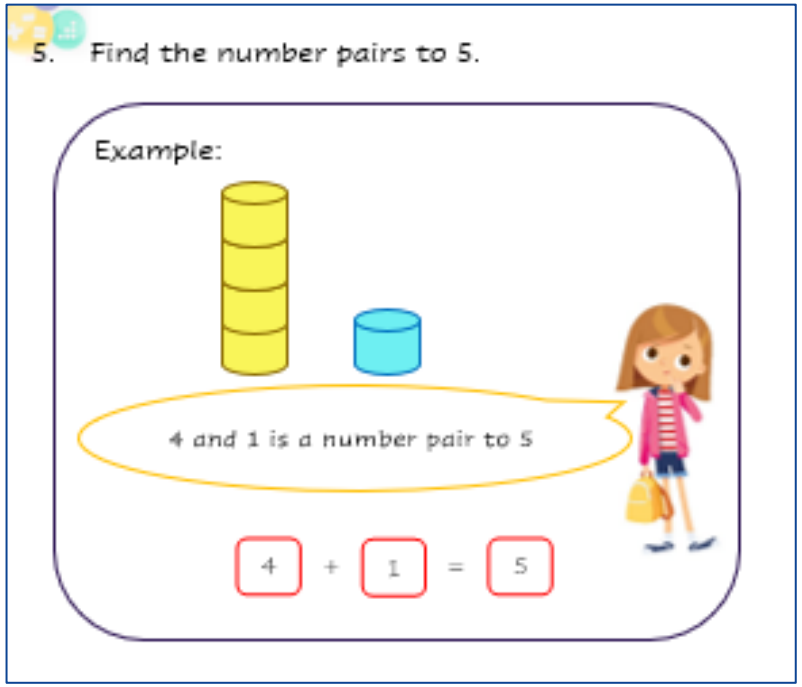
Three levels of representation

1. Concrete - use 3D objects to represent mathematical situations

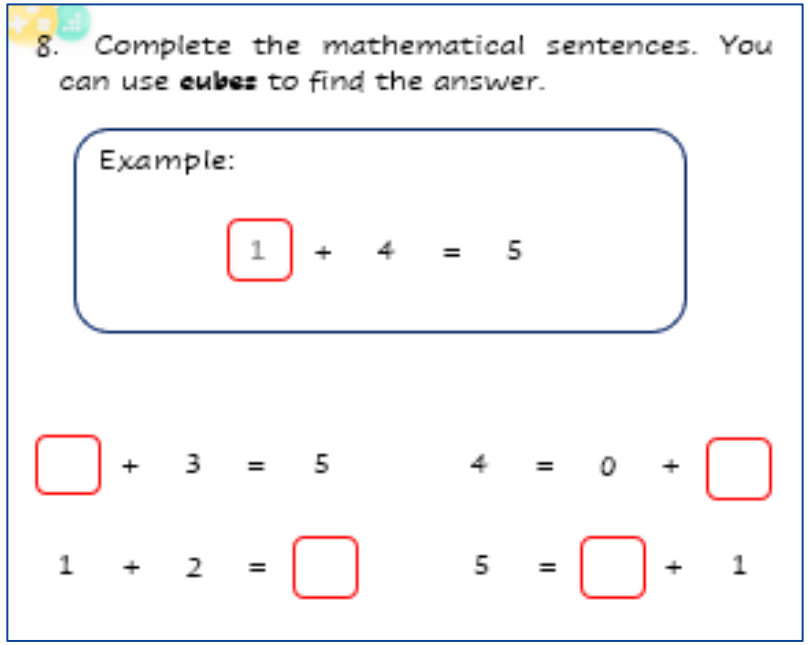
Use cubes to find the number pairs to 5.

Green Cubes		Red Cubes		
	+		=	5
	+		=	5
	+		=	5
	+		=	5
	+		=	5
	+		=	5

2. Pictorial - use pictures and diagrams



3. Abstract - use of mathematical symbols



Clear-cut Learning Objective (1)

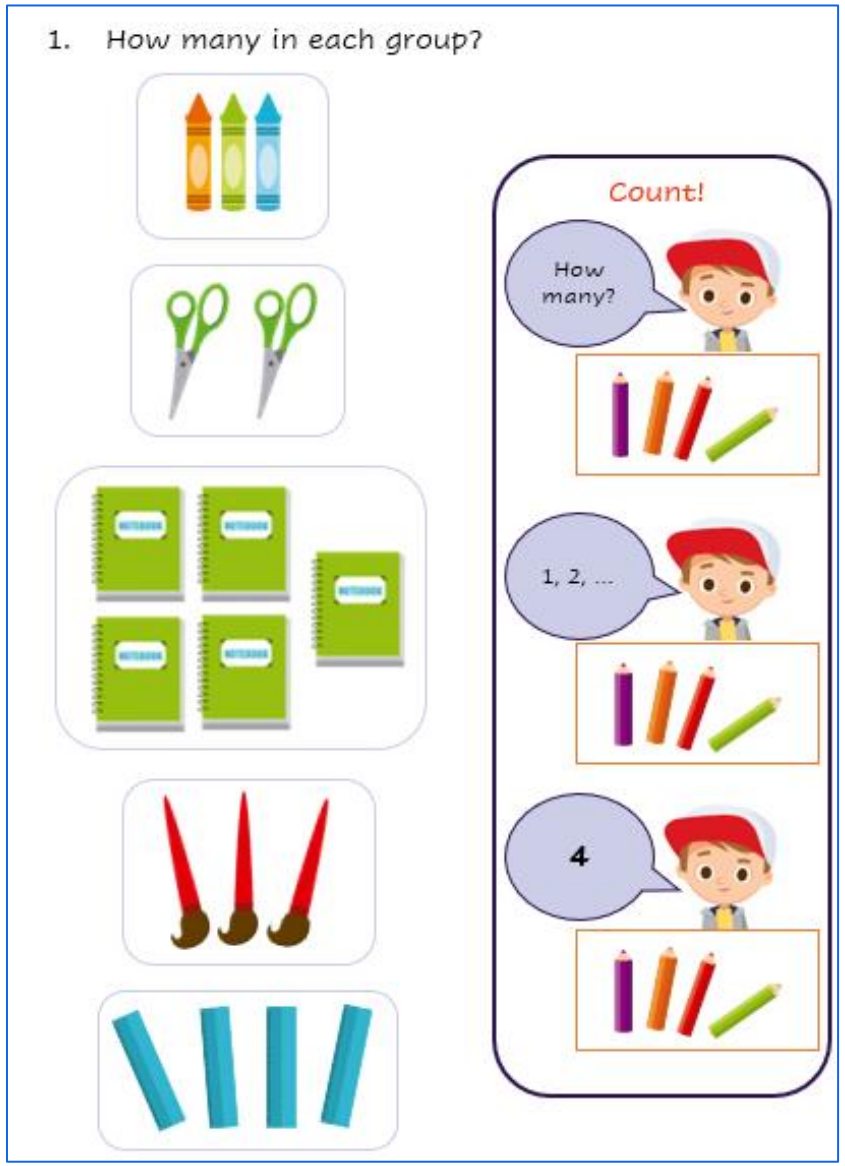
- The teaching material define clearly the concept, the procedure and the respective learning objective.

Lessons...

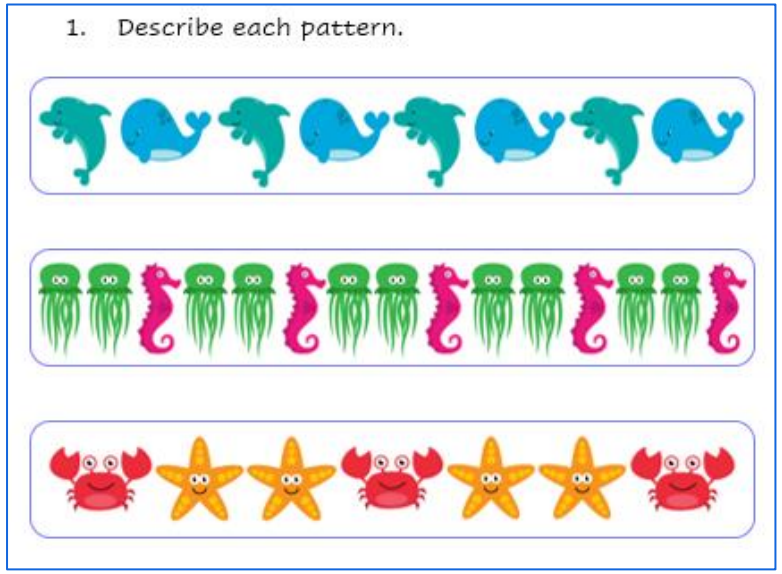
Concept: Numbers and quantities 1-5

Procedure: Measure quantities 1-5

Objective: Students count quantities 1-5.



Clear-Cut Learning Objectives

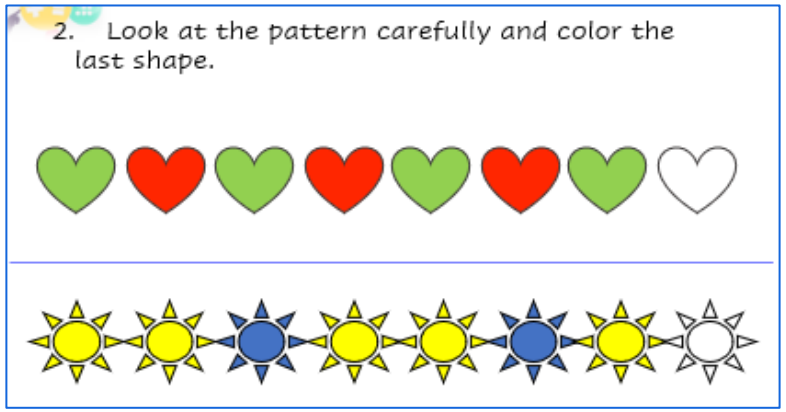


Lessons 1, 2, …

Concept: Pattern

Process: Visual pattern recognition

Objective: Students recognize visual patterns.



Lessons...

Concept: Pattern

Procedure: Visual pattern extension

Objective: Students expand visual patterns.

Examples from everyday life

- The mathematical concept and process under emphasis is presented through **examples from everyday life** and the student's familiar environment so that mathematics becomes meaningful.

Lessons...

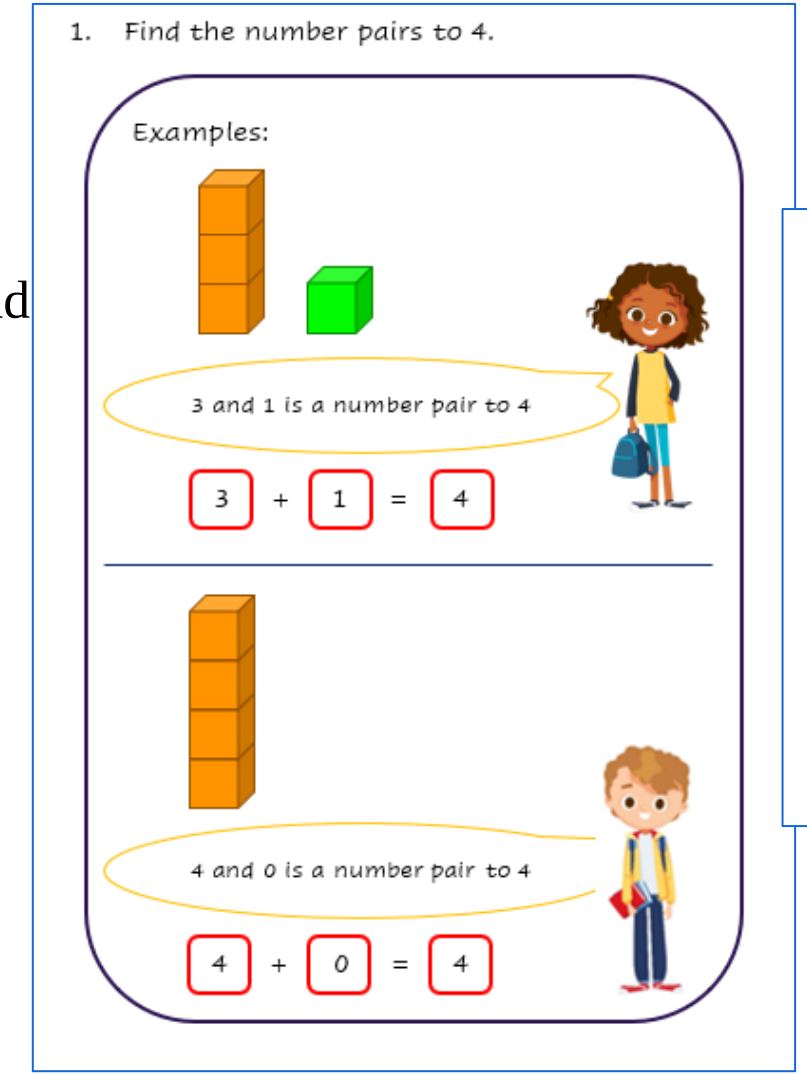
Concept: Additive structure problems

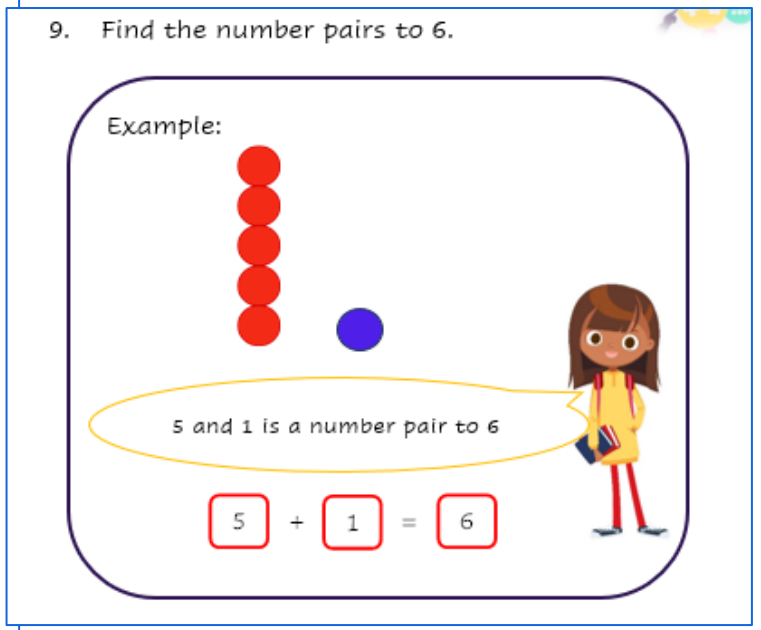
Procedure: Representing additive structure problems with mathematical propositions of intent

Objective: Students write mathematical addition sentences based on a mathematical story

Multiple examples

- **Multiple examples** are presented and then students are asked to apply the procedure.
- Time is provided to consolidate and practice the new knowledge.

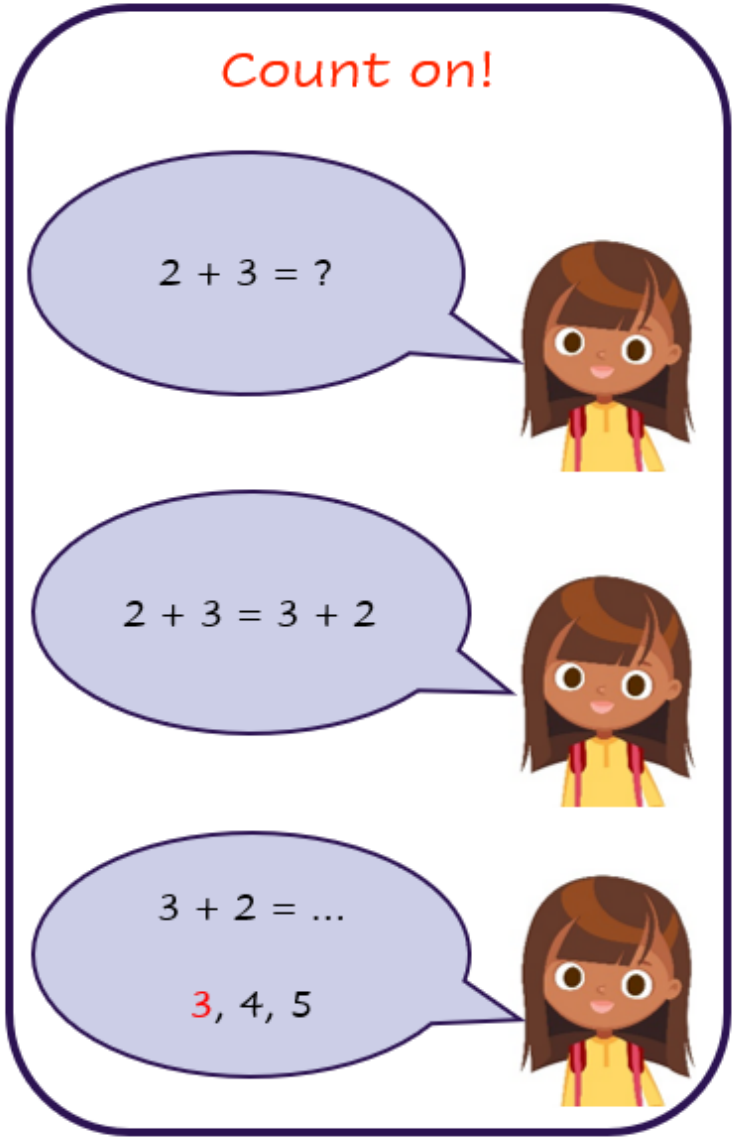




Questioning

Helpful questions (scaffolding) are presented to guide students in understanding the procedures.

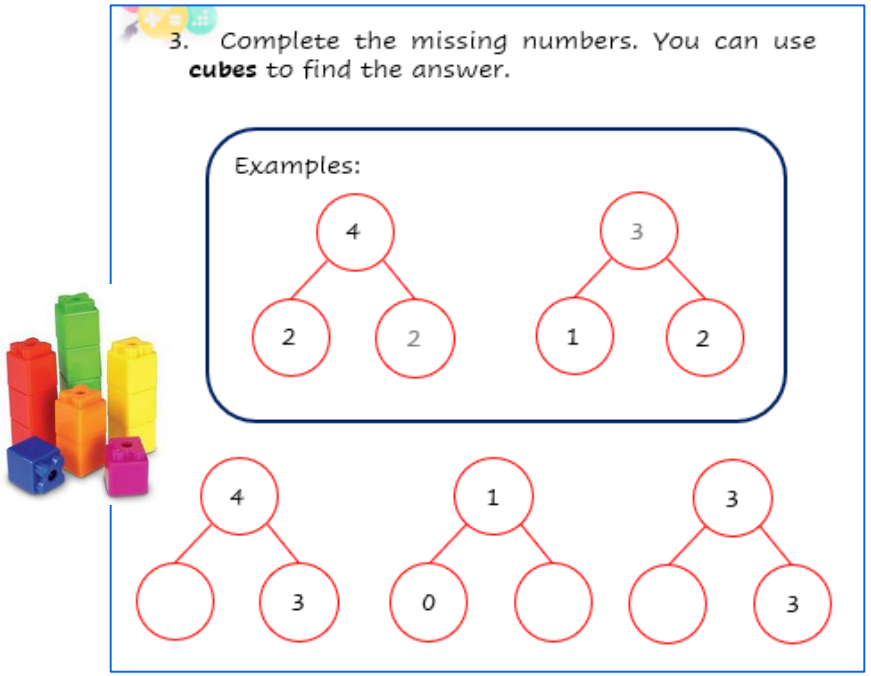
- ✓ Why did the girl write 2+3=3+2?
- ✓ Was her thinking correct?
- ✓ Would it be helpful to calculate 3 + 2 instead of 2 + 3?
- ✓ Why did she start counting from 3?
- ✓ Do you agree with what shed did?

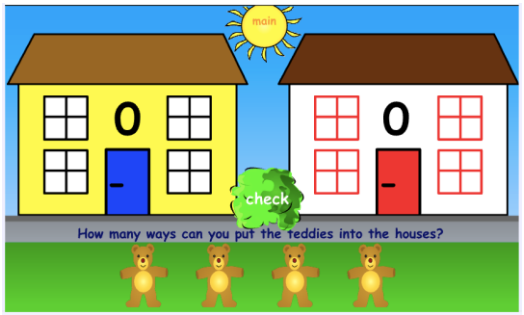


Mathematical tools and technology

The teaching material in printed form is enriched with:

- ✓ the use of **objects – mathematical tools** to represent basic concepts and processes.
- ✓ the **utilization of technological tools**, which are offered to introduce concepts, present procedures or practice.





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Lessons...

Concept: Division of 4

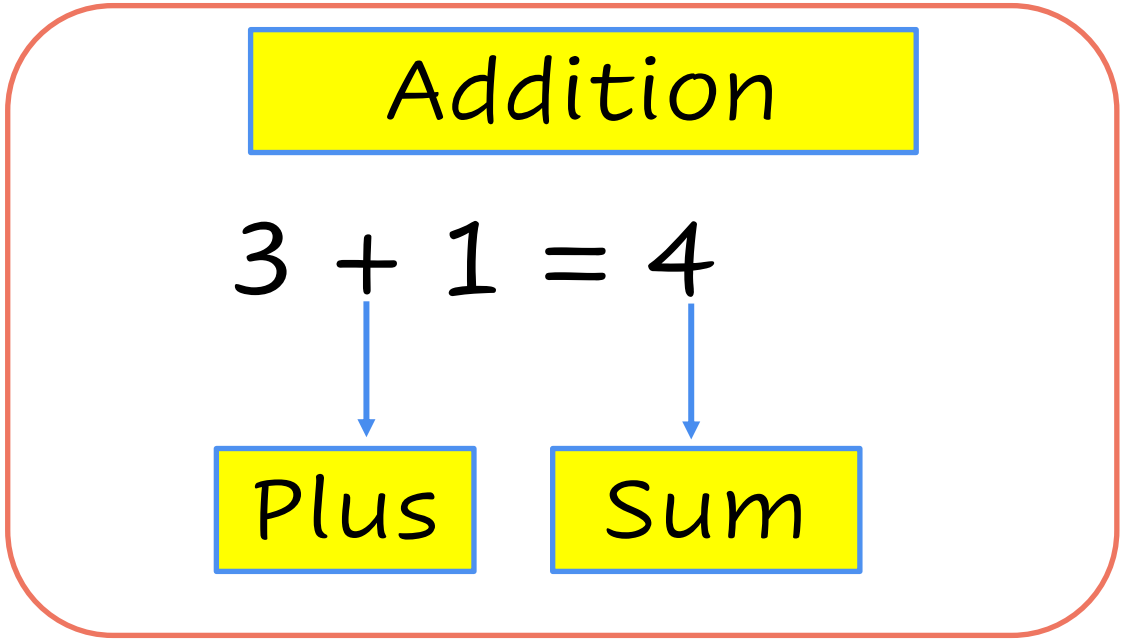
Procedure: Analysis and synthesis of 4

Objective: Students analyze and compose number 4 in all possible ways.

Mathematical terminology

- The teaching material is enriched with visual aids that present **the mathematical terminology** related to the basic concepts and processes.

- Important mathematical terminology (basic mathematical vocabulary) is presented through visual material that is posted on the classroom board and is used as a reference point during teaching and revision.



Systematic repetition

- Systematic repetition of terms, concepts and procedures is critical.
- For example, spending 10 minutes for practicing taught concepts / procedures in the beginning of each lesson.
- Practice using games (e.g. dominoes, bingo, dice).

4 + 3	4 - 3
7+ 1	7 - 1
4 + 2	8 -6

Use  to find the result	Use  to find the result
Use  to find the result	Pose a mathematical problem which can be solved with this mathematical sentence.
Use  to find the result	Make a plan to find the result.

Reflection

- Students are asked to describe their lesson objective and reflect at the end of each lesson on what they have learned.
- They are encouraged to “think aloud”. This forces them to work at a slower pace and refine their process.
- They are encouraged to ask questions and discuss their mistakes.
- Students are encouraged to work in pairs and discuss their work with each other.

Assessment

- It is important to make systematic, continuous, small, **formative assessments** that address specific learning objectives.
- Depending on the results of the formative assessment, the content of the following lessons is adjusted.

Teaching Approaches

Teaching approaches for the following topics

- Patterns
- Number sense
- Addition and subtraction
- Problem-solving
- Composing and decomposing
- Addition and subtraction strategies

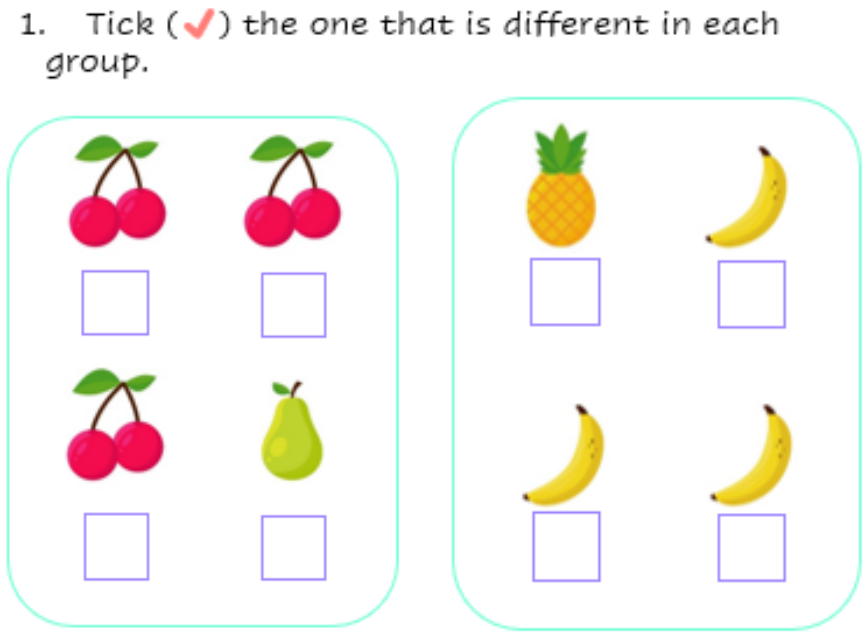
Patterns

Patterns

1. Finding differences
2. Finding similarities
3. Grouping objects
4. Recognizing patterns
5. Extending Patterns
6. Finding the rule
7. Constructing patterns

Finding differences

- First, we present groups of objects in which one of the objects differs from the others. All of the objects belong to the same category (e.g. fruits).
- Then, we present groups of objects in which one of object differs by one characteristic (e.g. colour). E.g. 3 blue cubes – 1 red cube
- We ask questions such as:
  - **"Which cube has a different colour?"**



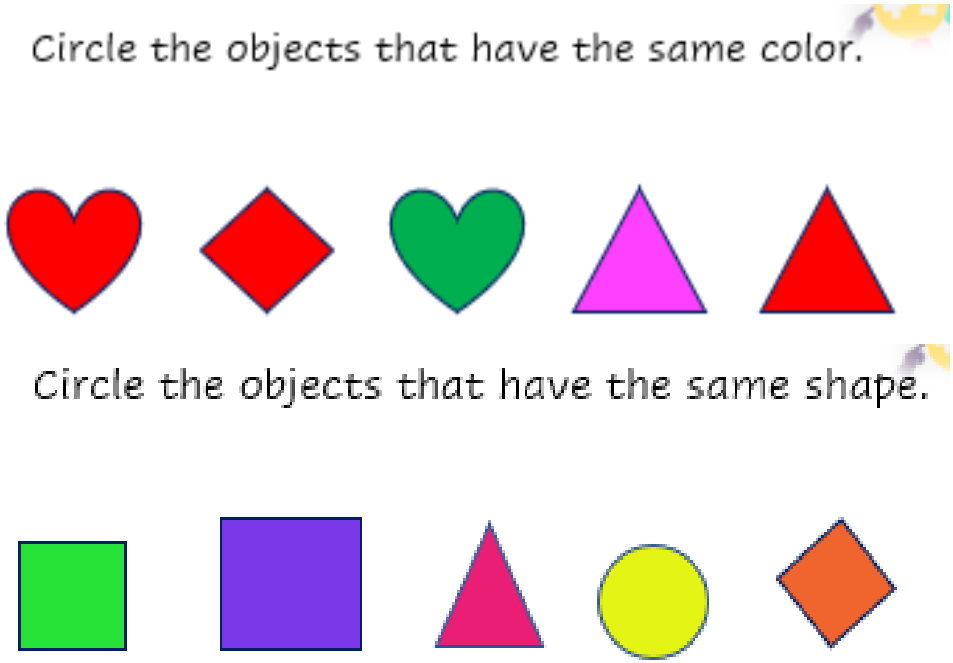
Finding similarities

- We present small groups made of different objects, some of which share a common characteristic

  E.g. same color, same shape, same size.

- We ask questions such as:

  - **“Which objects have the same colour?”**
  - **“Which objects have the same shape?”**
  - **“Which objects are the same size?”**



Grouping objects (1)

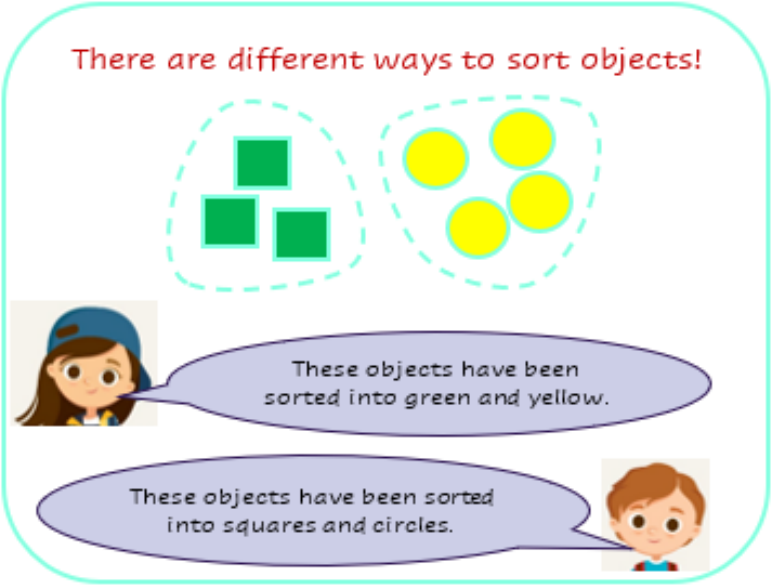
- We introduce students to small groups of objects that belong to the same category and can be grouped by color, shape, or size.

  E.g. green squares and red circles

- Students group the objects by choosing the grouping criterion.

- We ask questions such as:

  - **“How could the shapes be grouped?”**
  - **“Can they be grouped into greens and reds?**
  - **“Can they be grouped based on their colour?”**
  - **“Can they be grouped into circles and squares?”**



Tell how the objects can be grouped.

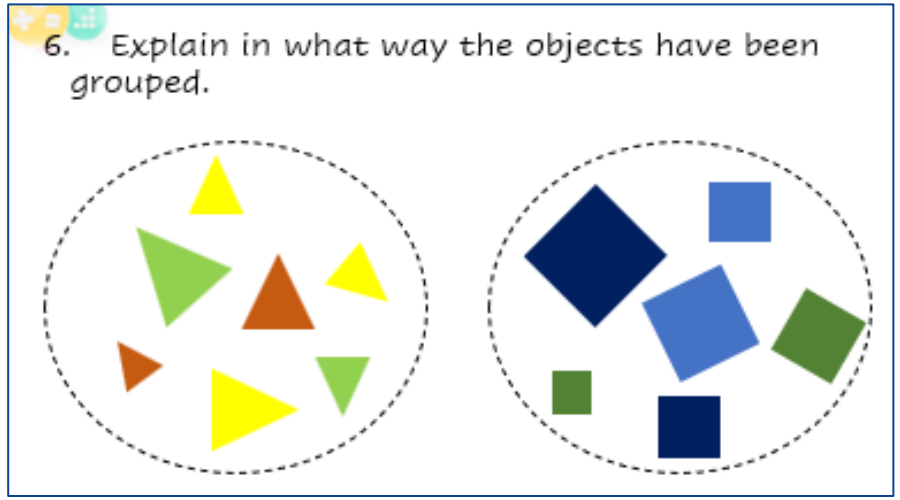
Grouping objects (2)

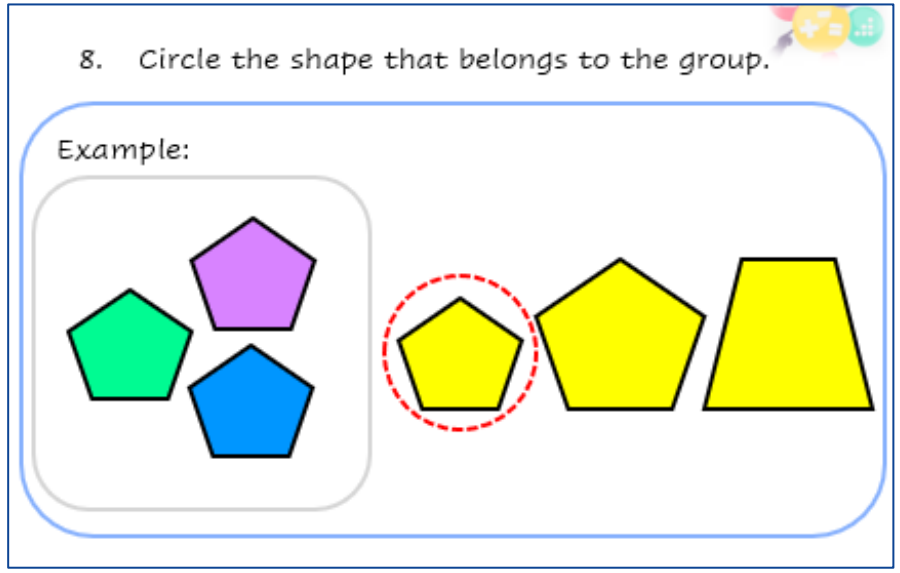
- We present students with small groups that have a common characteristic: same color, same shape, or same size. Students are asked to identify the grouping criterion.

  E.g. triangles of different size and shape

- We ask questions such as:

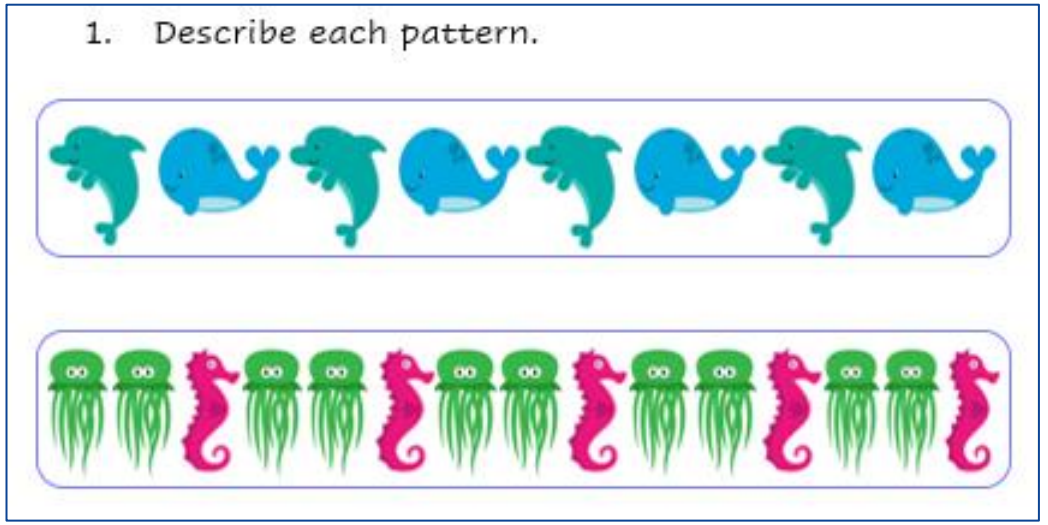
  - **“In what way were the objects grouped?”**
  - **“What do the shapes in this group have in common?...Which shape belongs to this group?”**





Recognizing patterns

- First, we present repeating patterns made of two terms (AB) and students are asked to describe the pattern and recognize the repetition of these terms.
- Then, we introduce repeating patterns with different structures e.g. ABC, AAB.



Extending patterns

- Students are asked to “read” the pattern and find the next terms (extend it). We ask questions such as:

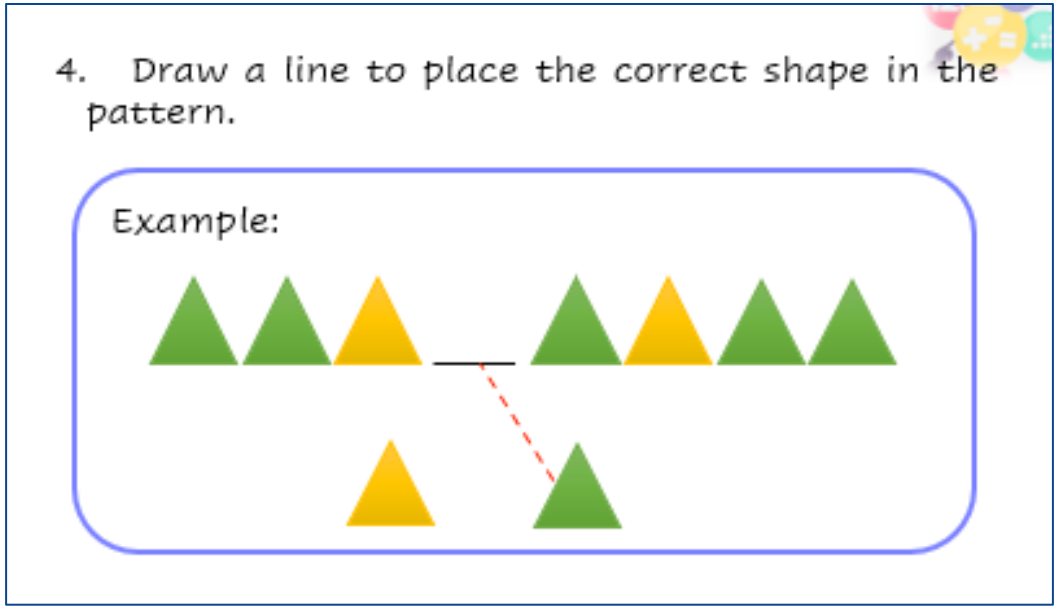
  - **“Here there is a dolphin (we show the first term), a whale, a dolphin etc. Can you continue?”**
  - **“What do you notice about the pattern? ”**
  - **“What would be the colour of the next heart in the pattern?”**

Finding the rule

- We introduce patterns AAB, BBA, ABB, BAA, ABC

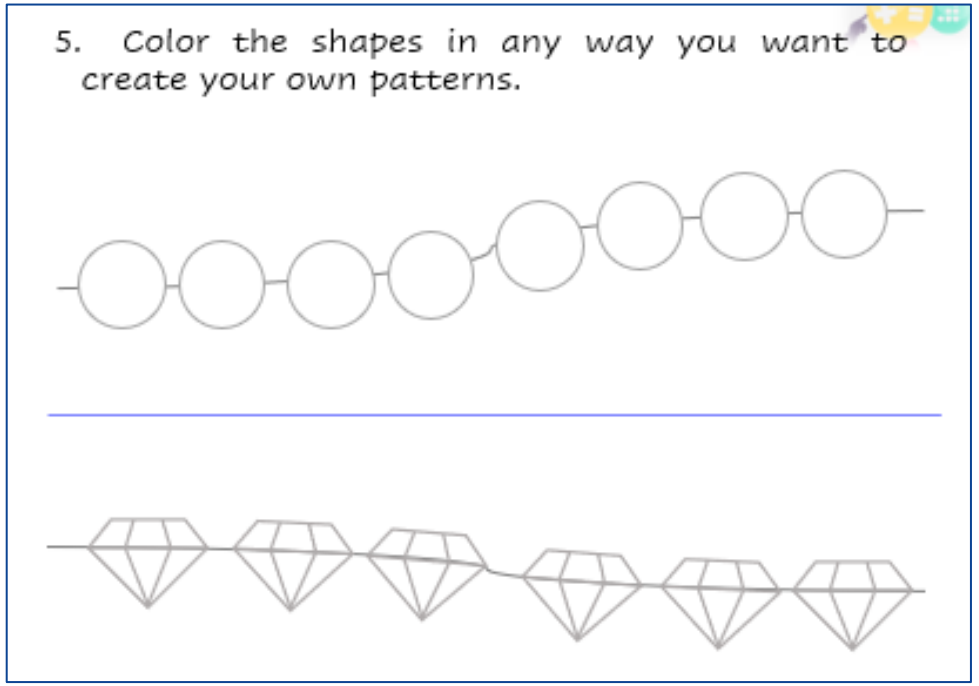
  E.g. yellow-yellow-blue, red-blue-blue, yellow-blue-red

- We ask questions such as:
  - **“What is the rule of the pattern?”**
  - **“What would be the colour of the missing triangle?”**



Constructing patterns

- Finally, we give students the opportunity to construct their own patterns.
  - **“How do you make the pattern? … You start with the color blue, then red, red, blue… Now what color would you say should come next?”**
  - **“What is the rule of the pattern you have constructed?”**



Number sense

Number sense

1. Counting up to 10 with objects
2. Counting up to 10 with pictures
3. Number sense up to 10
4. Representation of numbers up to 10
5. Writing numbers up to 10
6. Sorting – Comparing numbers

Counting numbers up to 10 with objects

- We start with small numbers (first 1-5 and then 6-10).
- We present different groups of objects so that students come to realize that there is a 1-to-1 correspondence, that is, we only count each object once.
- We constantly associate the process of counting with the question: **“How many?”.**
- After presenting several examples, we invite students to list different groups of objects in the same way.
- We encourage students to describe their thinking, using complete sentences:
  “One, two, three, four...There are four cartons.”

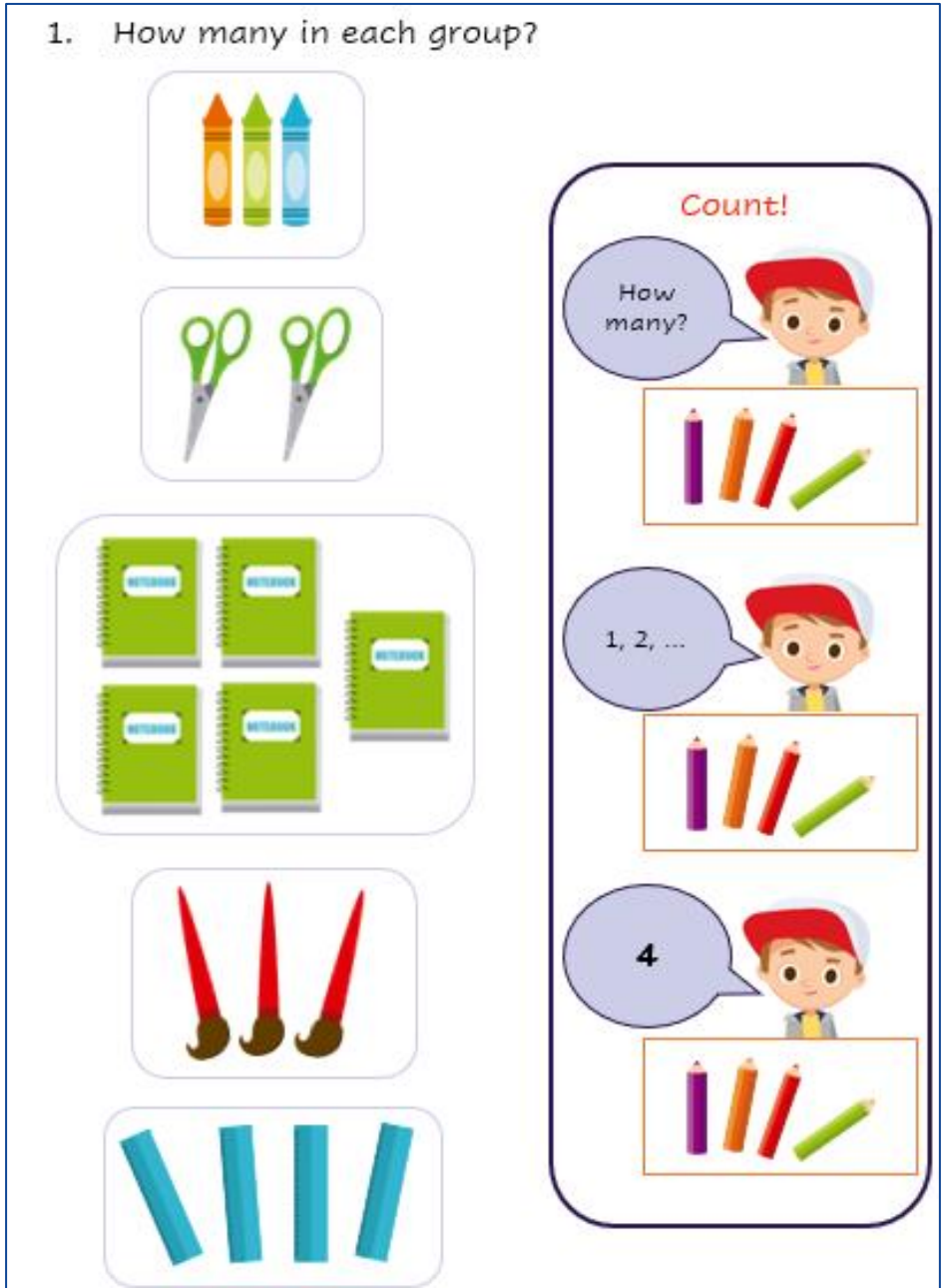
1 2 3 4

1 2 3 4 5

Counting numbers up to 10 with pictures

- We present students with different groups of objects in pictures.
- We invite students to count them by describing their thinking in complete sentences:

“One, two, three...There are three brushes.”

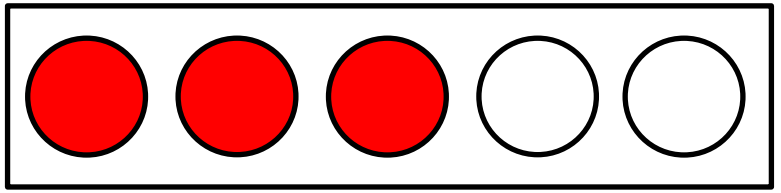


Number sense up to 10 (1)

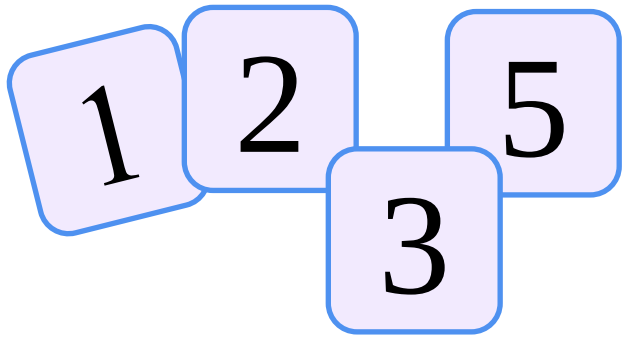
- We start with small numbers (first 1-5 and then 6-10).
- To connect the verbal word with the quantity and the symbolic form of the number, we present:
  - ***Distinct objects***
  - ***Grid of 5 (for numbers up to 5) and grid of 10 (for numbers up to 10)***
  - ***Cards with the symbolic form of the numbers***
- We count the distinct objects, colour in the grid as many dots as the set of objects and select the tab with the corresponding number.

Real Materials

Pictures

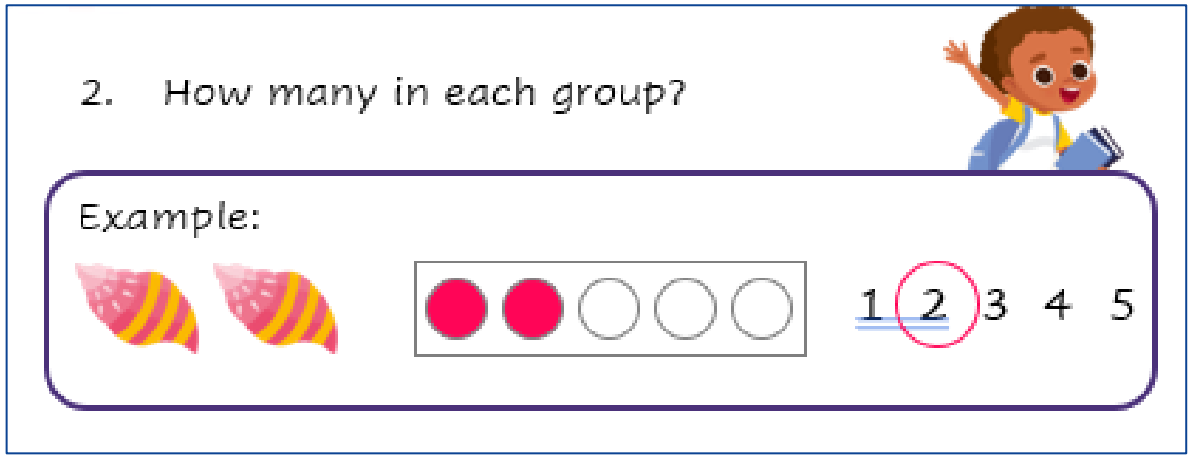


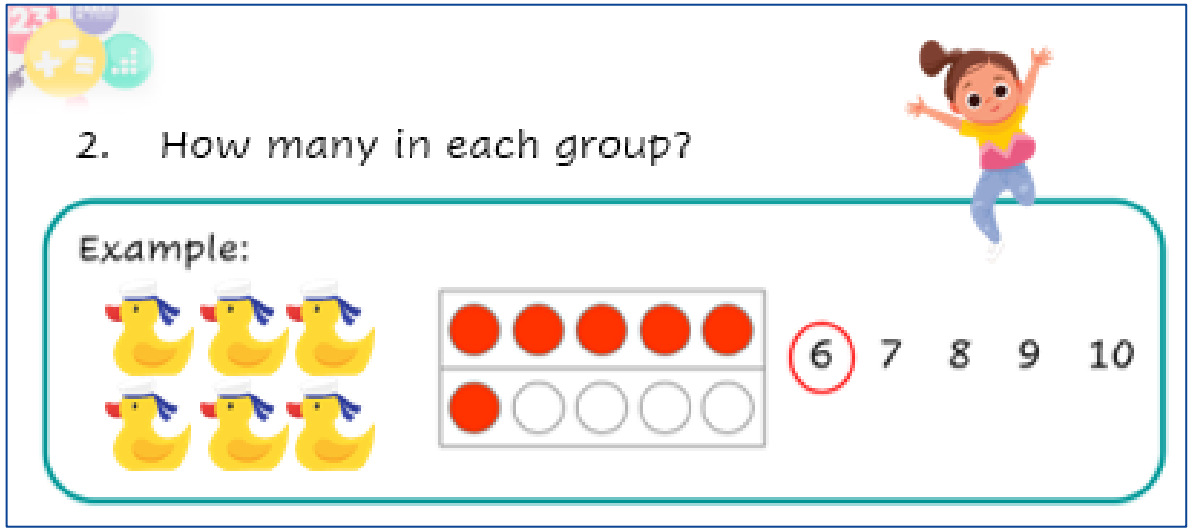
Symbols



Number sense up to 10 (2)

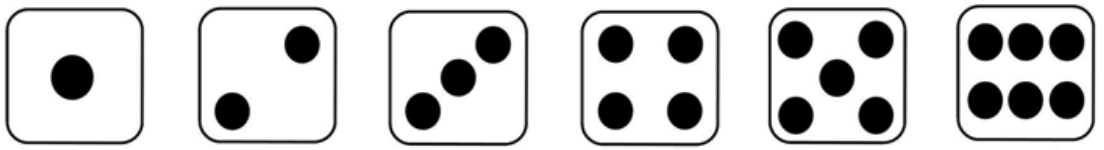
- We provide multiple representations such as:
  - ***Images with discrete objects***
  - ***Grid of 5 (for numbers up to 5) and Grid of 10 (for numbers up to 10)***
  - ***Numbers in symbolic form***
- We count the distinct objects, colour in the grid as many dots as the set of objects and choose the symbol that corresponds.

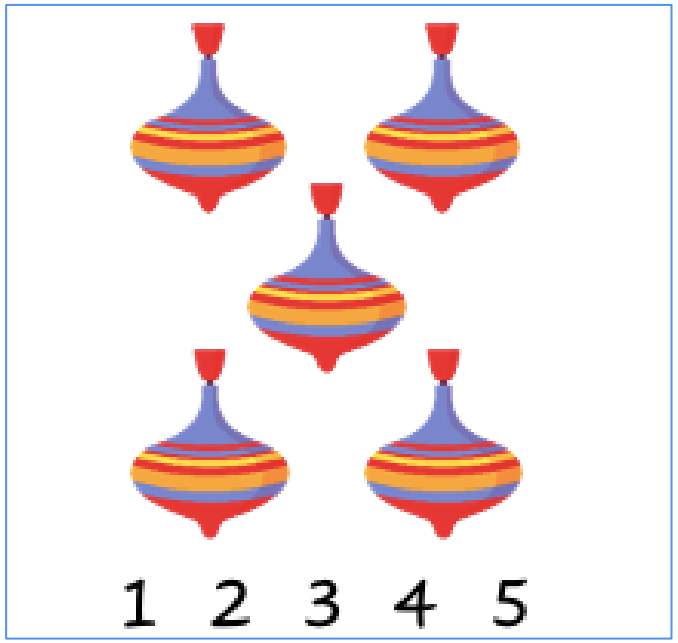




Number sense up to 10 (3)

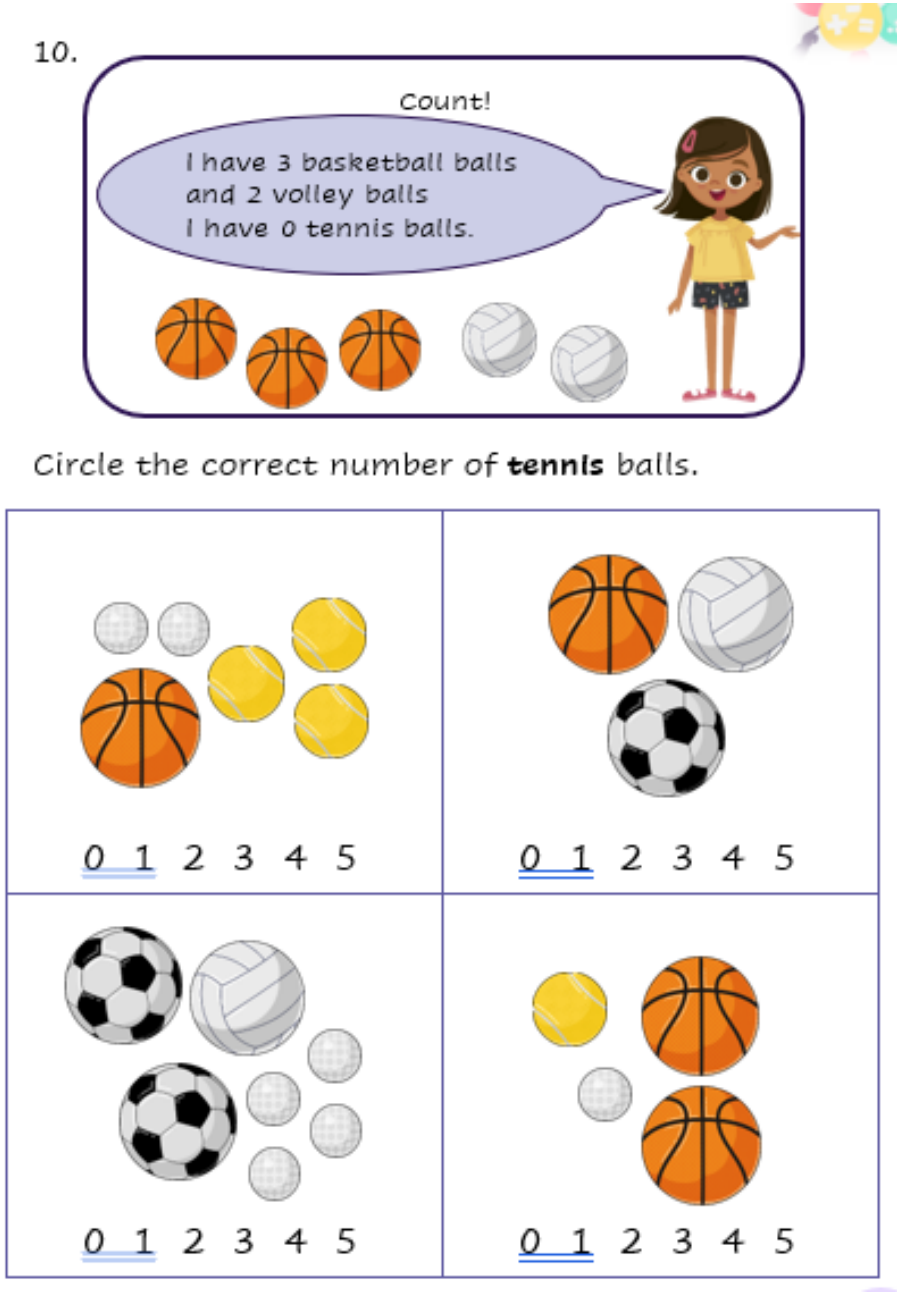
- We present students with pictures in which discrete objects that are organized in a way that represents the structure of the number. In this way, students are encouraged to subitize (avoid counting 1-1)
- Dice or dominoes could be used.
- We ask questions such as:
  - ***"Is it possible to say how many without counting?"***





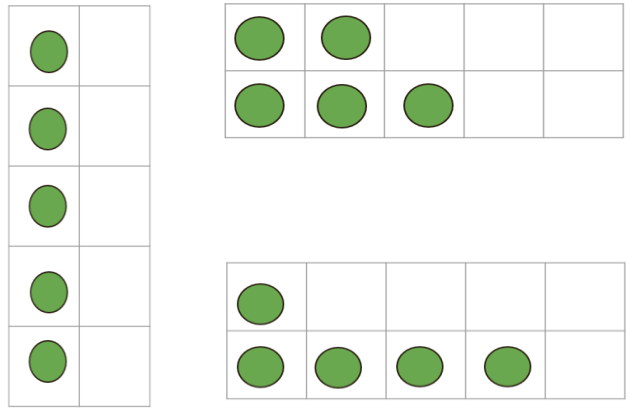
Number sense up to 10 (4)

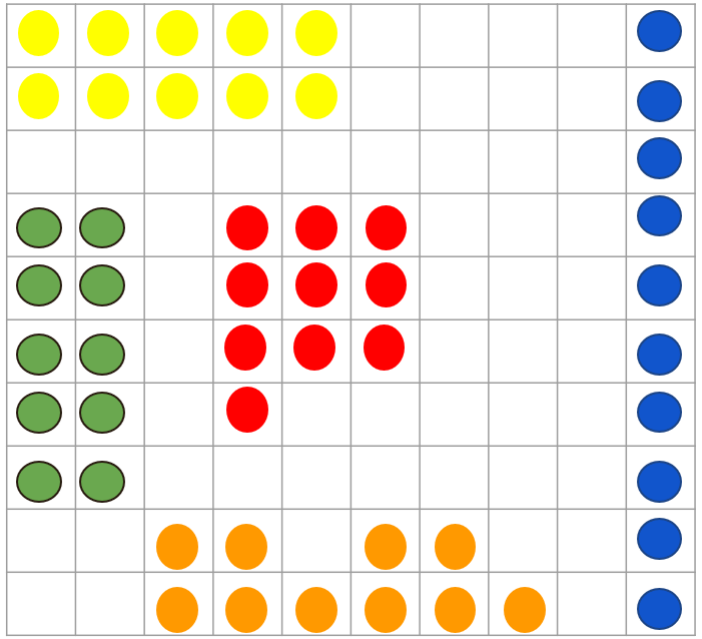
- We present groups of objects that belong to the same category (e.g. different balls, different flowers)
- We ask questions such as:
  - **“How many tennis balls are there?”**
  - **“How many basket balls are there?”**
  - **“How many volley balls are there?”**
  - **“How many are all the balls?”**



Representation of numbers up to 10

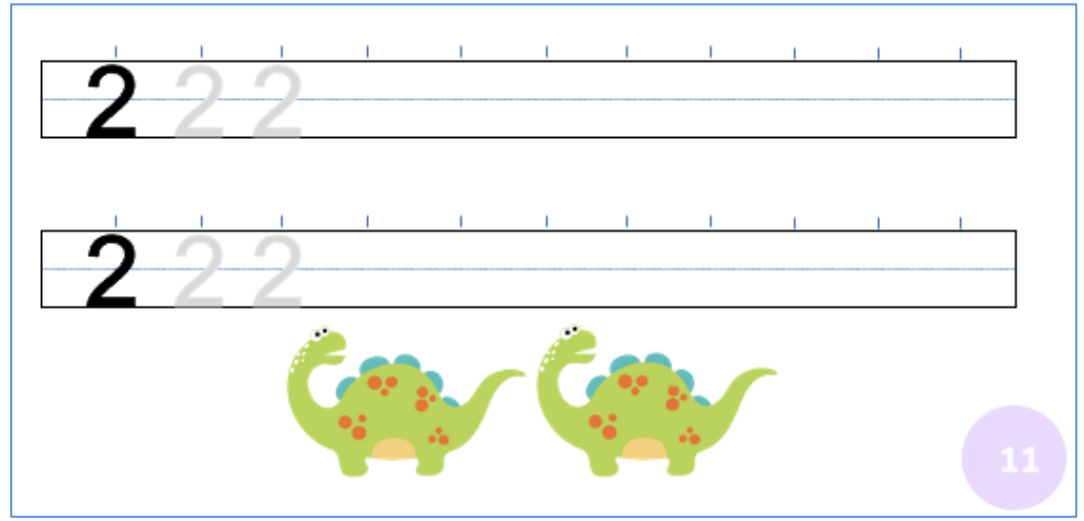
- Representing numbers helps students in developing fluency and flexibility about the structure of numbers.
- Introduction to Addition
  - We ask questions such as:
    - **"In what ways can you draw 5 dots?“**
    - **"In what ways can you draw 10 dots?"**

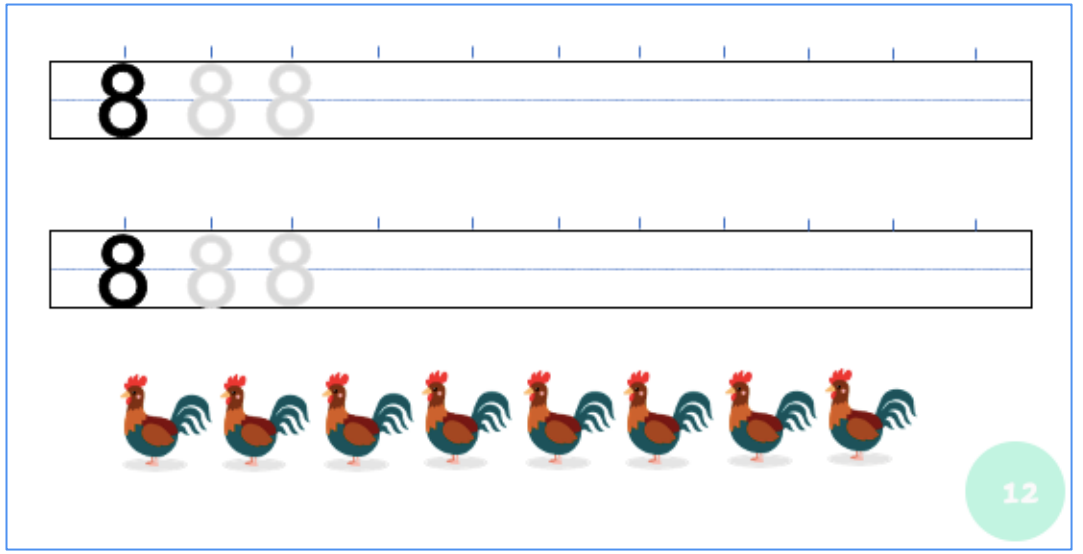




Writing numbers up to 10

- We provide time for children to practice using structured writing exercises.
- We present the symbolic and the pictorial representation of the number in order to give meaning to the symbol.

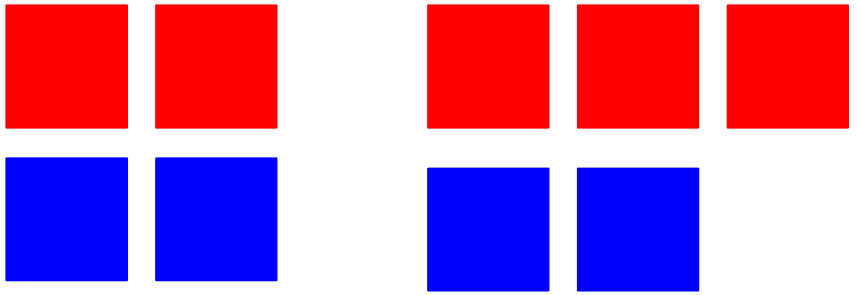




Sorting – Comparing numbers (1)

- Matching two sets of objects 1-1 helps students compare the two quantities to decide if they are equal or not

  E.g. check if there are as many red squares as blue



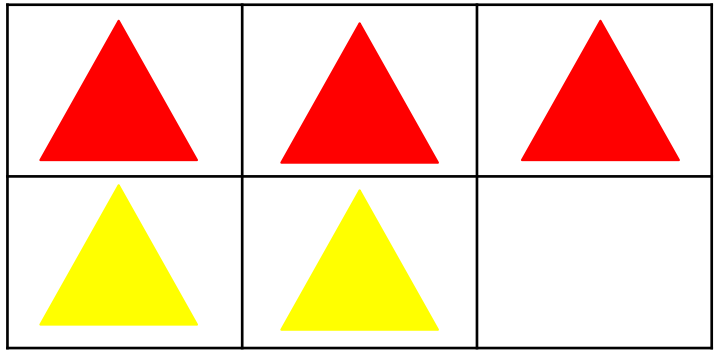
- Distinct objects
- Objects in order
- We ask questions such as:
  - **“Are there as many red cubes as blue cubes?”**
  - **“Is there a red cube for every blue cube?”**
  - **“Are there as many erasers as pencils?”**
  - **“Is there a pencil for every eraser?”**

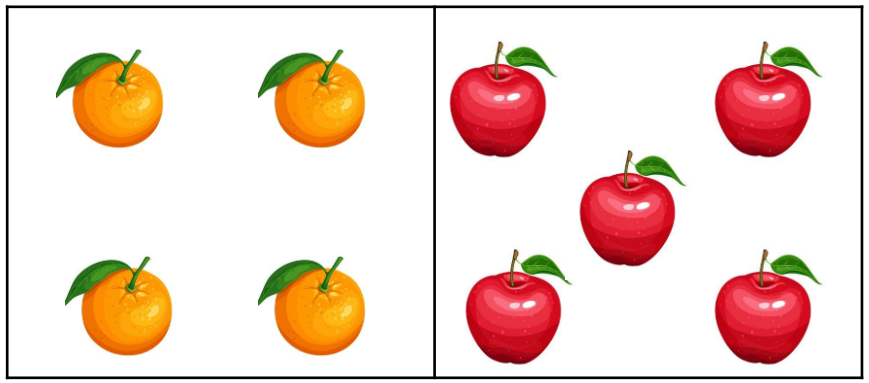
Number classification and comparison

- Matching 1-1
- Multiple Representations
- Use of grid
- We ask questions such as:
  - **“Are there as many kites as children?”**
  - **“Is there a kite for every child?”**
  - **“Are there as many umbrellas as children?”**
  - **“Is there an umbrella for every child?”**

Number classification and comparison

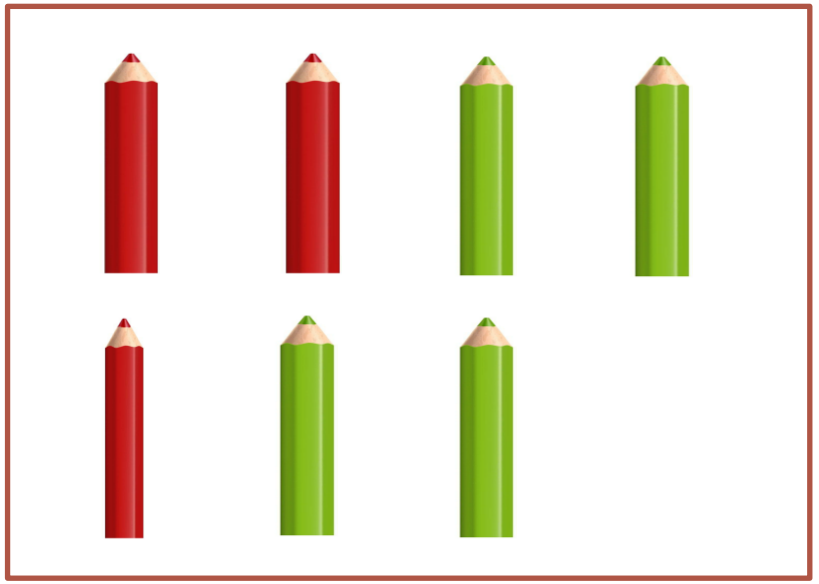
- Students compare the two quantities and determine which group has the most or the least objects.
- Distinct groups
- Different colour- type
- We ask questions such as:
  - **“How many red triangles are there?”**
  - **“How many yellow triangles are there?”**
  - **“Which are the most/least triangles the red or the yellow?”**





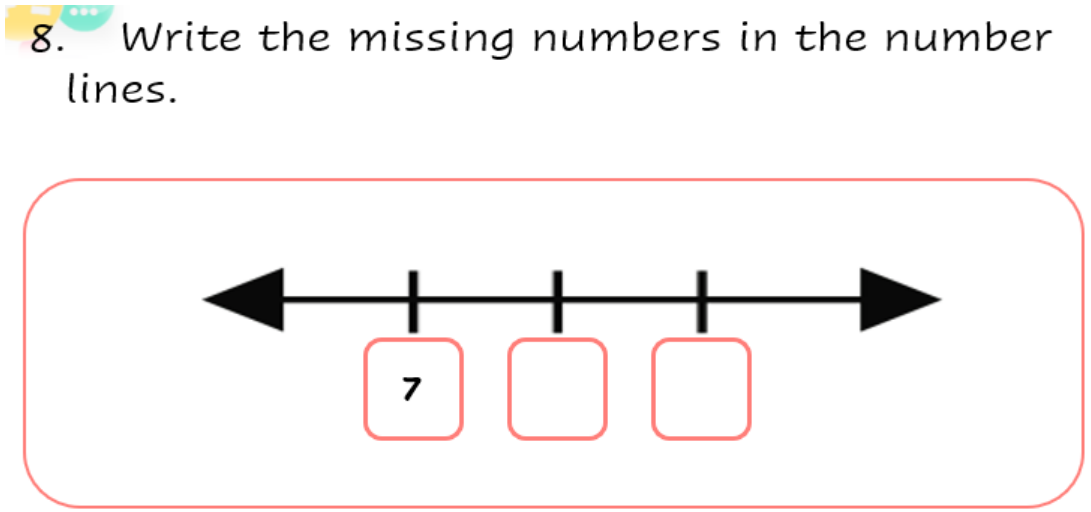
Number classification and comparison

- We present students with a group of objects that belong to the same category (e.g., fruits) and ask them to decide which subgroup has the most or the least objects.
- We ask questions such as:
  - **“How many red pencils are there?”**
  - **“How many green pencils are there?”**
  - **“Are the red pencils more than the green pencils, or are they the same?”**



Number classification and comparison

- Students compare numbers to find the smallest one or the largest one.
- Write numbers in the correct order.
- Use a number line.



Addition and subtraction

Addition and Subtraction

1. Addition
2. Addition Stories - Aggregation
3. Addition Stories - Augmentation

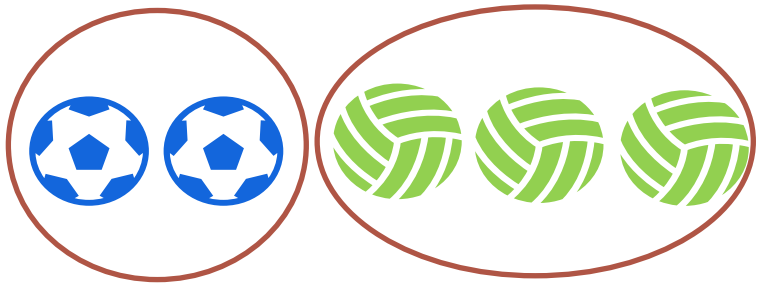
4.Subtraction

5. Subtraction Stories

6. Addition-Subtraction

Addition

- First, students are introduced to addition using manipulatives.
- The manipulatives are organised in ways that the two addends (the two parts of the whole) are shown separately (e.g. in rings) and then they could be moved together.

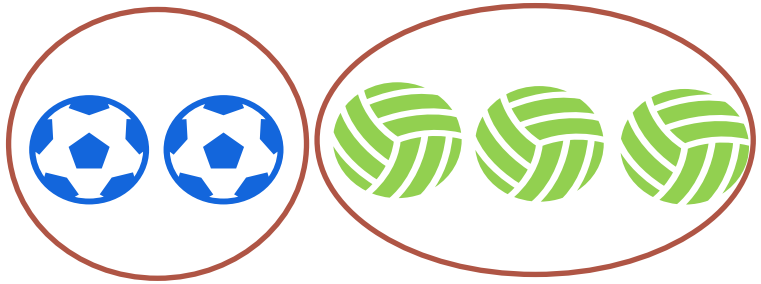


2 + 3

3 + 2

Addition

- The two addends are written in symbolic form, without presenting the sum.
- We introduce the + sign and name it as "plus". At the same time, we mention the word "add".
  - **“There are 2 soccer balls and 3 volleyballs.”**
  - **“We write this as 2 plus 3, which means I add 2 and 3.”**
  - **“2 shows 2 soccer balls, and 3 shows 3 volleyballs.”**

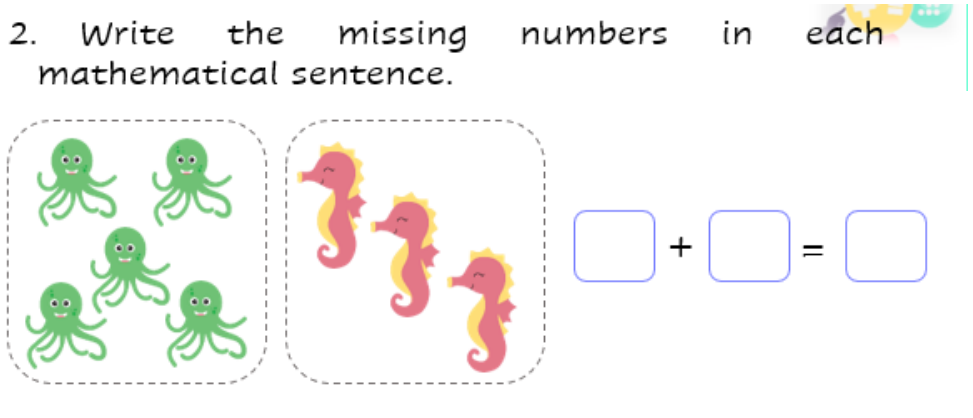


2 + 3

3 + 2

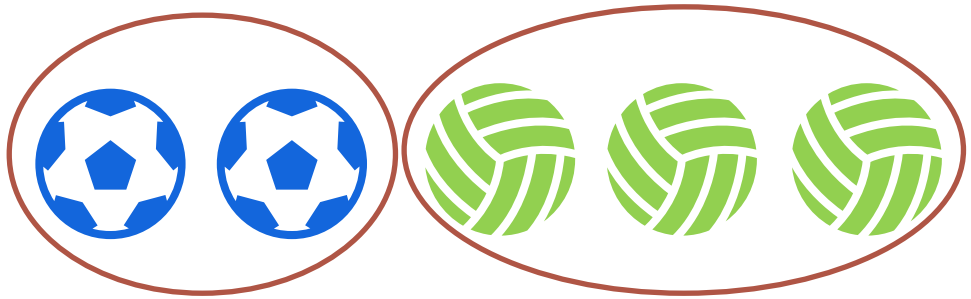
Addition

- We invite students to write mathematical addition sentences based on pictures that clearly show the two parts to be added.
- We ask questions such as:
  - **“How many are the goldfish?”**
  - **“How many are the dolphins?”**
  - **“How many are all the fish?”**



Addition

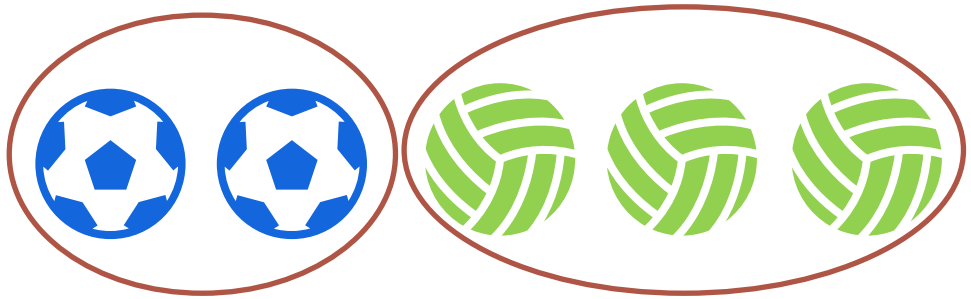
- Then we introduce the = sign and call it "equals" to show the equality between the whole and the sum of the two parts.
- Initially, we refer to the whole and note that it is possible to divide it into 2 parts
  - **“5 is equal to 2 and 3” or “5 is equal to 3 and 2”**



$$5 = 2 + 3$$
$$5 = 3 + 2$$

Addition

- Finally, we present 4 different mathematical sentences:
  - **“2 soccer balls and 3 volleyballs equal 5 balls.”**
  - **“3 volleyballs and 2 soccer balls equal 5 balls.”**
- We provide many examples to the students, and then we present them with objects and pictures.
- At the end, we ask students to write four different mathematical addition sentences.



$$5 = 2 + 3$$
$$5 = 3 + 2$$

Addition Stories: Aggregation

- We present simple stories, which are gradually translated into mathematical addition sentences.
- We combine pictures with short questions asking students to identify the quantities involved in a mathematical addition sentence.
- In the beginning, stories involve grouping situations.
- We ask questions such as:
  - **“How many animals are outside the river?”**
  - **“How many animals are in the river?”**
  - **“How many are all the animals?”**

1 + 4 = 5

Addition Stories – Augmentation

- Stories are enriched with situations undergoing change.
- We emphasize the words "first," "then," and "now."
- We ask questions such as:

- **First: “How many children were on the bus first?”**
- **Then: “How many children entered the bus next?”**
- **Now: “How many are all children on the bus now?”**

First

Then

Now

1 + 2 = 3

Subtraction

- Then we ask the students to write subtraction sentences based on pictures that clearly show the minuend and how it is reduced.

- We ask questions to complete the mathematical sentence on a symbolic level using words like "first", ”later", "now".

  - **Now: “How many lifejackets were there first?”**
  - **Then “How many lifejackets were lost later?”**
  - **Now: “How many lifejackets are left now?”**

Subtraction Stories

- We present simple stories, which are gradually translated into mathematical subtraction sentences.
- Stories are about situations undergoing change.
- We emphasize the words, “at the beginning", “later", “now".
- We ask questions such as:
  - **First: "How many apples were on the apple tree at the beginning?“**
  - **Then: "How many apples did John cut later?"**
  - **Now: How many apples are left now?“**

6 – 4 = 2

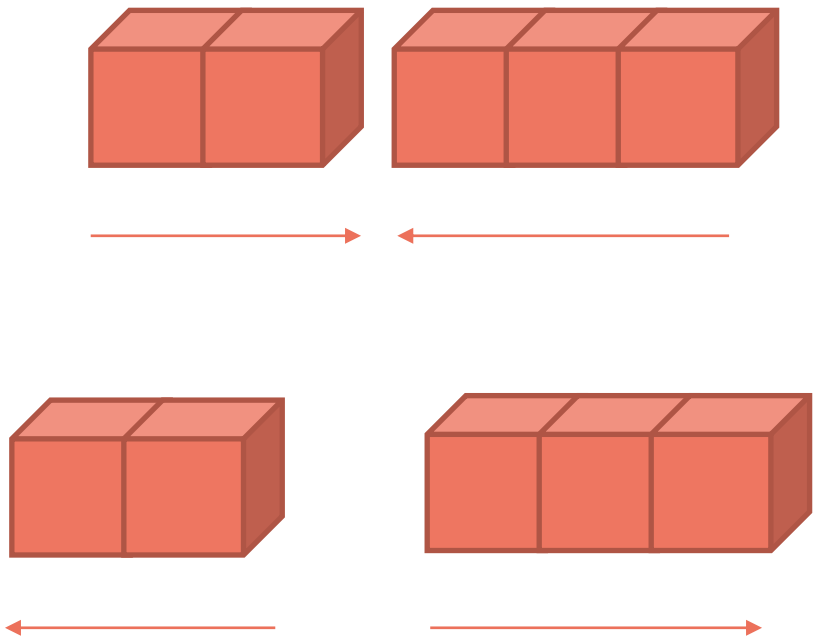
First

Later

Now

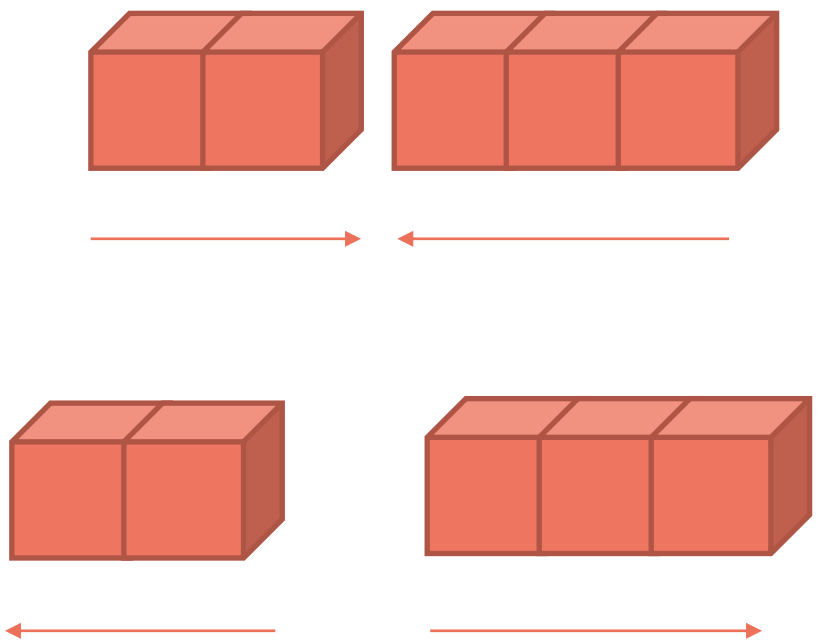
Addition- Subtraction

- Subtraction is linked to addition.
- We give the students a set of unifix cubes, separate them into two parts, and show them the movement of aggregating the two parts by emphasising the word "addition" and then partitioning them by emphasising the word "subtraction".
-



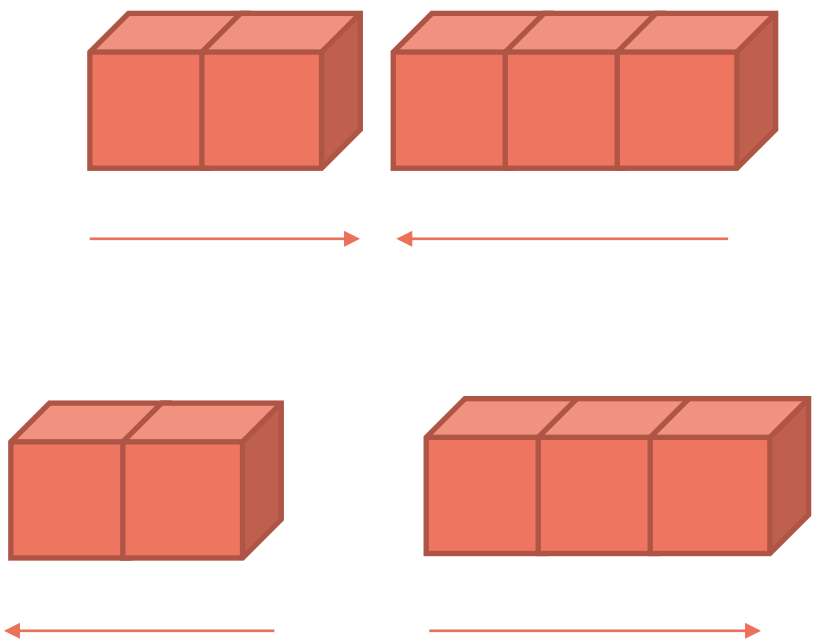
Addition- Subtraction

- We present stories that match the partitioning and the aggregation of the cubes.
- We ask questions such as:
  - **Aggregation: “I have 2 blue pencils. I bought 3 more pencils. How many pencils do I have now?”**
  - **Partitioning: “I had 5 pencils. I gave 2 pencils to my friend. How many pencils do I have left?”**



Addition- Subtraction

- We write mathematical sentences that correspond to each story:
  - **“I have 2 blue pencils. I bought 3 more pencils. How many pencils do I have now?”**
  - **2 + 3 = 5**
- We introduce the − sign and call it "minus".
  - **“I had 5 pencils. I gave 2 pencils to my friend. How many pencils do I have left?”**
  - **5 – 2 = 3**
- At the same time, we introduce the word “subtract”:
  - **“We subtract 2 from the 5 pencils. There are 3 pencils left. 5 minus 2 equals 3.”**



Problem solving

Problem Characteristics (1)

- **Description of Addition and Subtraction Mathematical Stories**
- **One-step problems**: We present problems that can be solved with just one mathematical sentence
- E.g. 7+2=

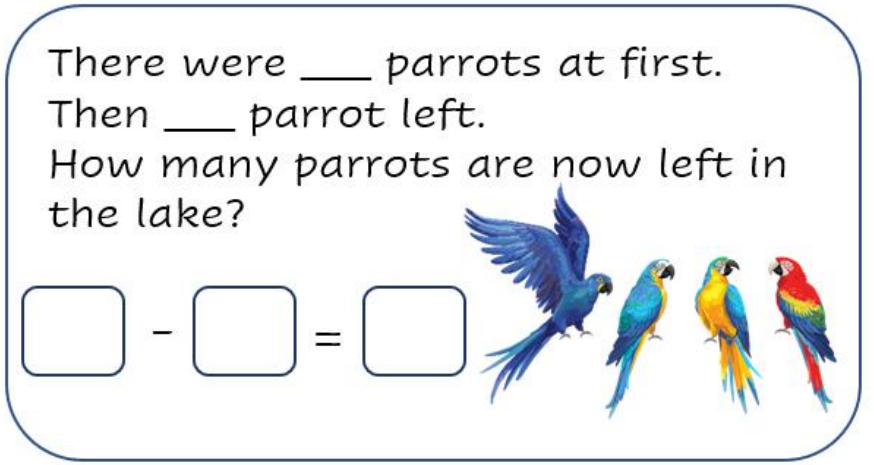
Anna ate 2 slices of pizza with pineapple and 3 slices of pizza with mushrooms. How many slices of pizza did Anna eat?

Mathematical sentence: _____

Answer: _____

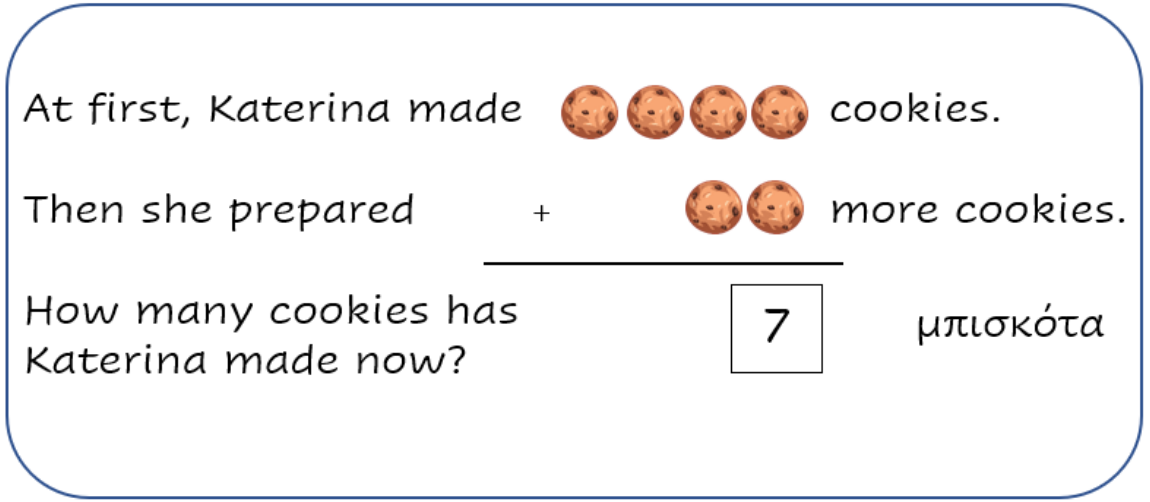
Problem characteristics (2)

- **Clear and short sentences:** We use problems with short sentences and simple vocabulary so that students can understand the content and solve the problem.
- **Virtual Representations:** We use virtual representations to demonstrate the problem so that students make connections between the pictorial and the symbolic representations.



Suggestion for presentation of the problems (1)

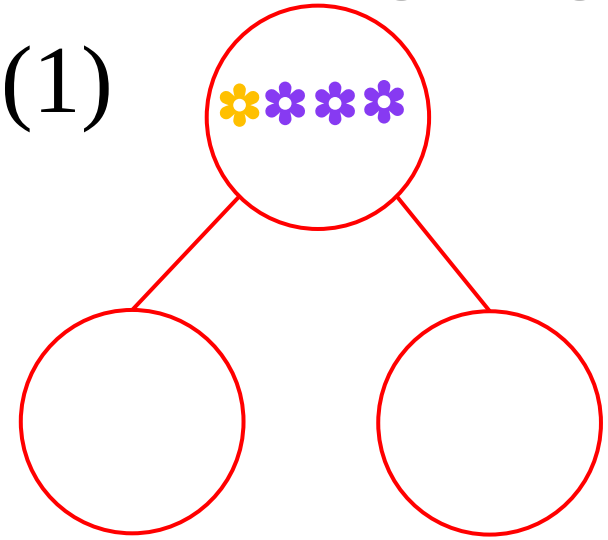
- **Introduction to vertical addition and subtraction**
- **Short verbal description:** We present students the opportunity to visualise the problem using less verbal description and simple vocabulary.
- **Making connections pictorial-symbolic representations:** We employ virtual representations to introduce the vertical addition and subtraction.

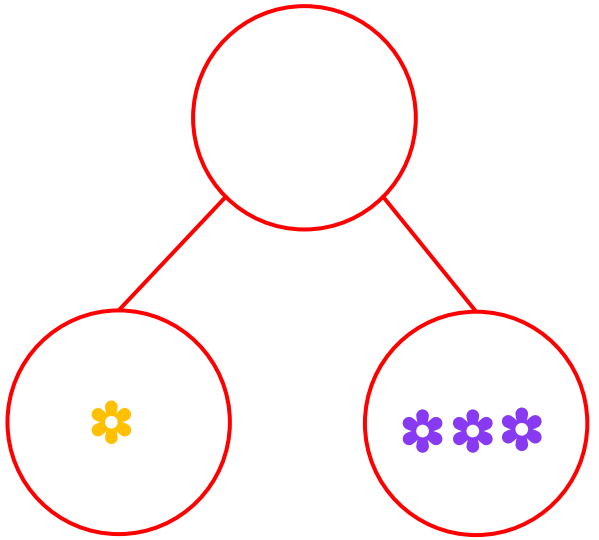


Composing and decomposing numbers

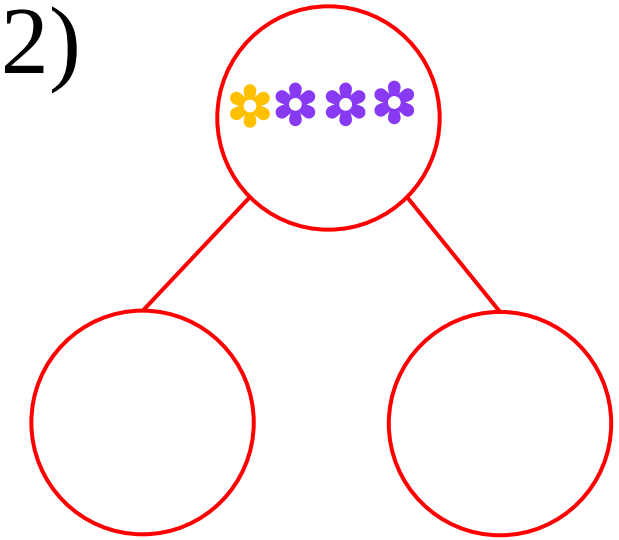
Composing and decomposing numbers (1)

- We introduce students into composing and decomposing numbers. Initially, we do not insist on finding on the possible ways.
- Students practice in composing and decomposing using manipulatives or pictures, as well as part-part-whole diagrams.

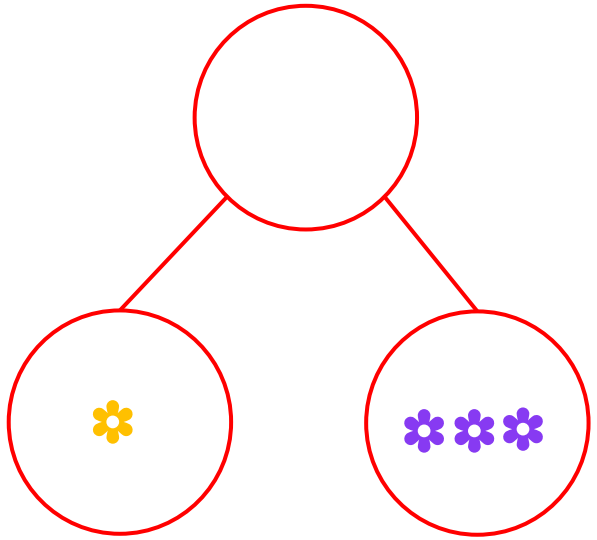




Composing and decomposing numbers (2)

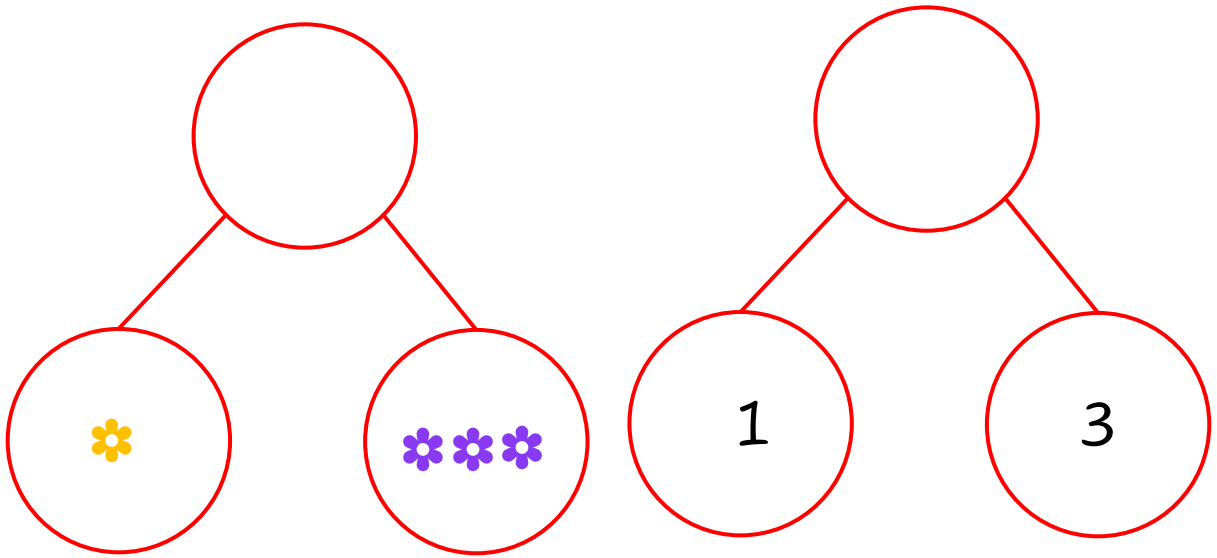


- Students are encouraged to separate the manipulatives into 2 parts and to describe in complete sentences the parts and the whole, sometimes starting from the whole and sometimes from the parts.
- The manipulatives should move from the whole to the parts (decomposition) and from the parts to the whole (composition) so that the students understand that the whole and the two parts represent the same quantity.



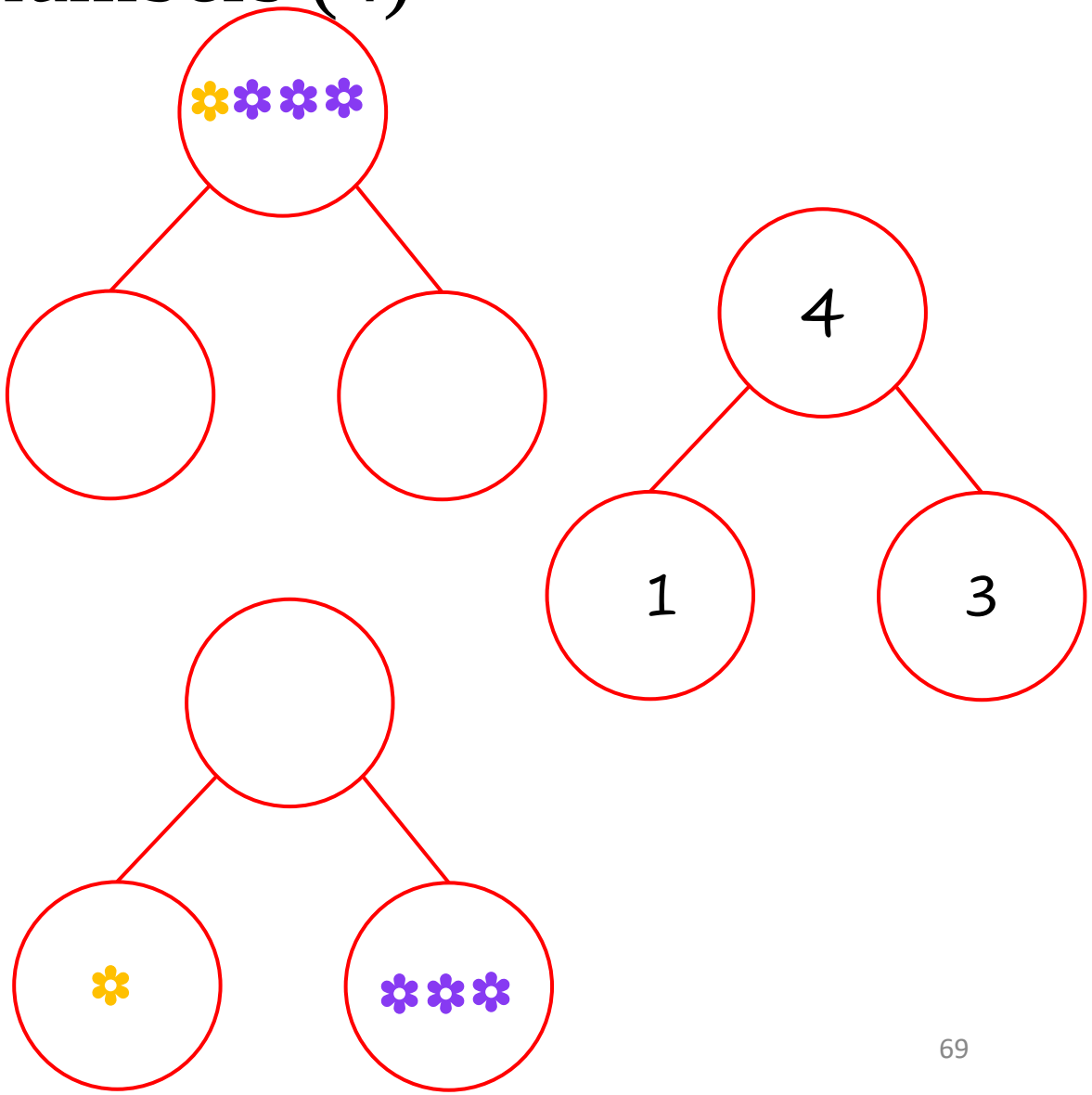
Composing and decomposing numbers (3)

- We describe in another grid the way in which the number has been analysed.
- The transition to the symbolic representation is usually difficult, so we aim to constantly make connections between then numbers in symbolic form with the quantities they represent.



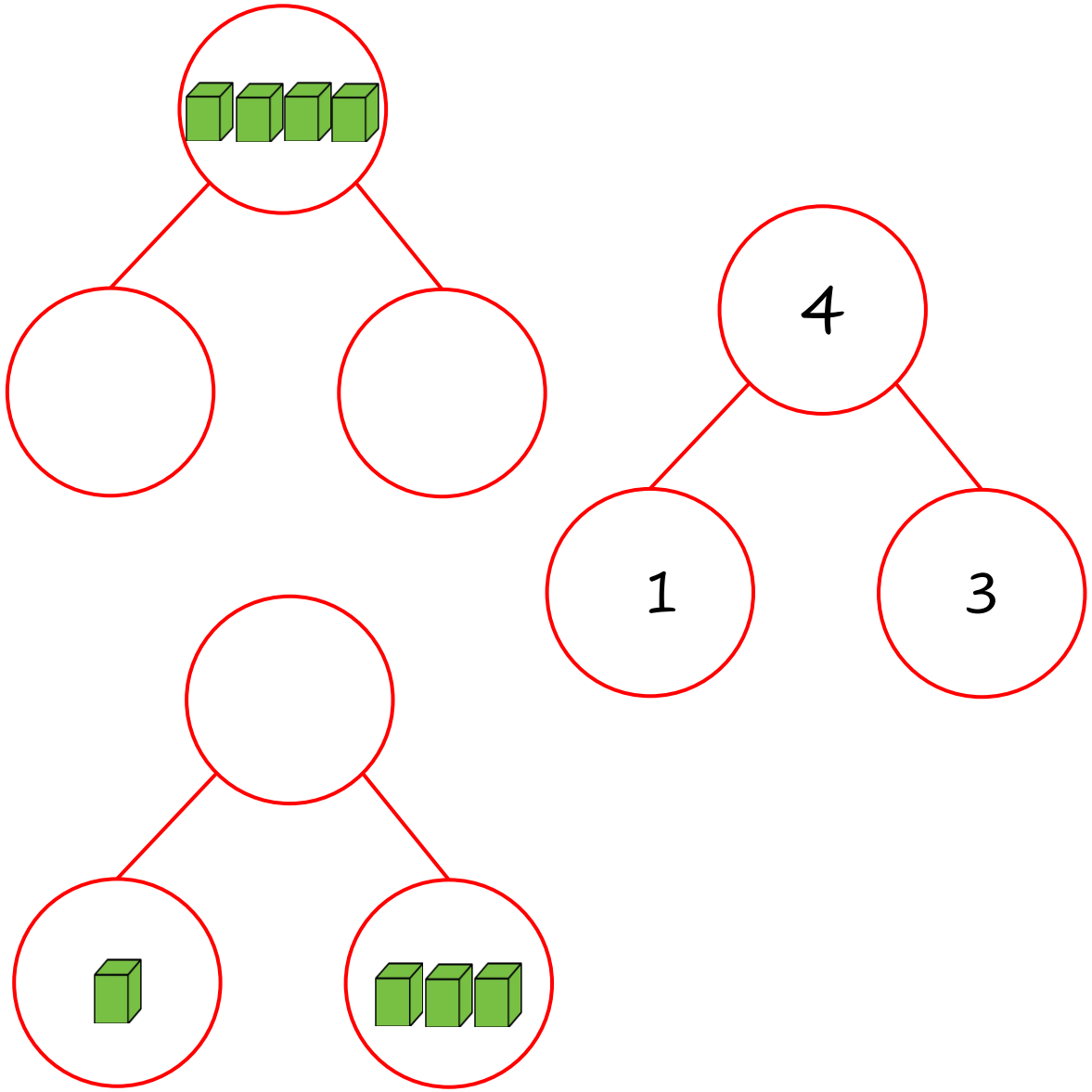
Composing and decomposing numbers (4)

- Students are encouraged to describe, using complete sentences, what each number represents.
  - **“4 shows the set of flowers”**
  - **“1 shows the number of yellow flowers”**
  - **“3 shows the number of purple flowers”**



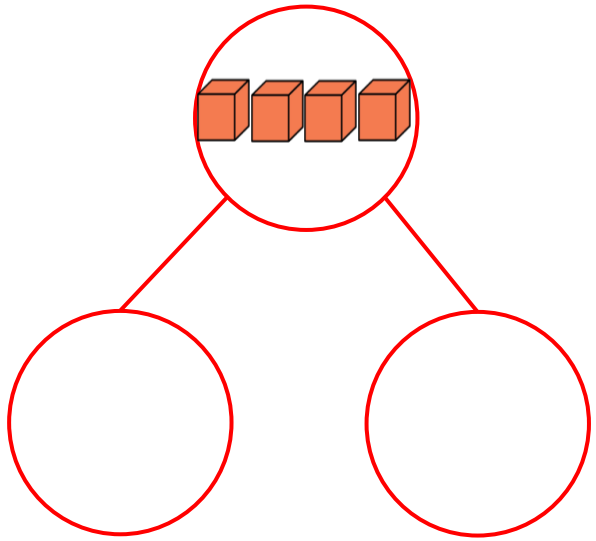
Composing and decomposing numbers (5)

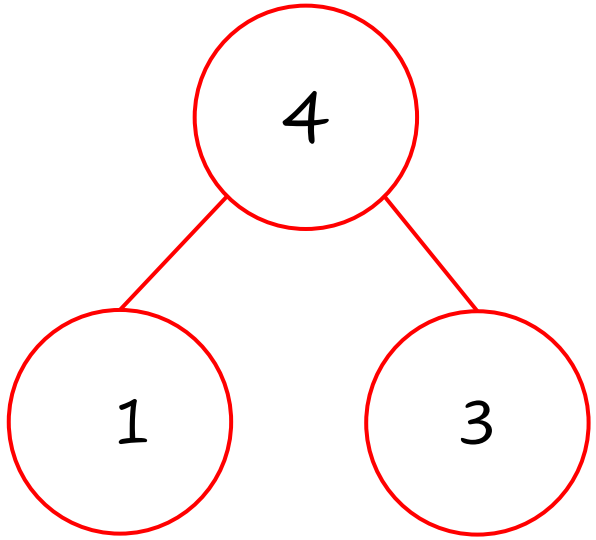
- Finally, students are encouraged to find all the possible ways they can decompose a number.
- At this stage, it is better for students to use cubes of the same size and color to focus their attention on how a group of cubes is decomposed into parts in different ways, regardless of other features, e.g., colour or size.

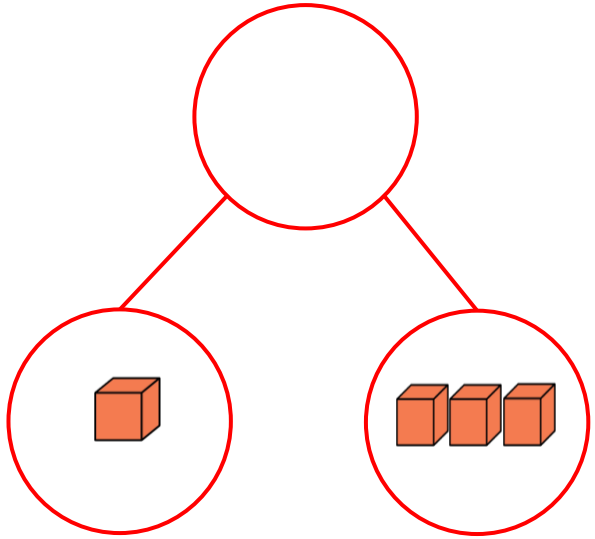


Composing and decomposing numbers (6)

- We ask questions such as:
  - **“What do you notice about the size of the parts and the size of the whole?”**
- We ask students to repeat the sentence each time:
  - **"____ is the whole ___ is part, and ____ is part.”**

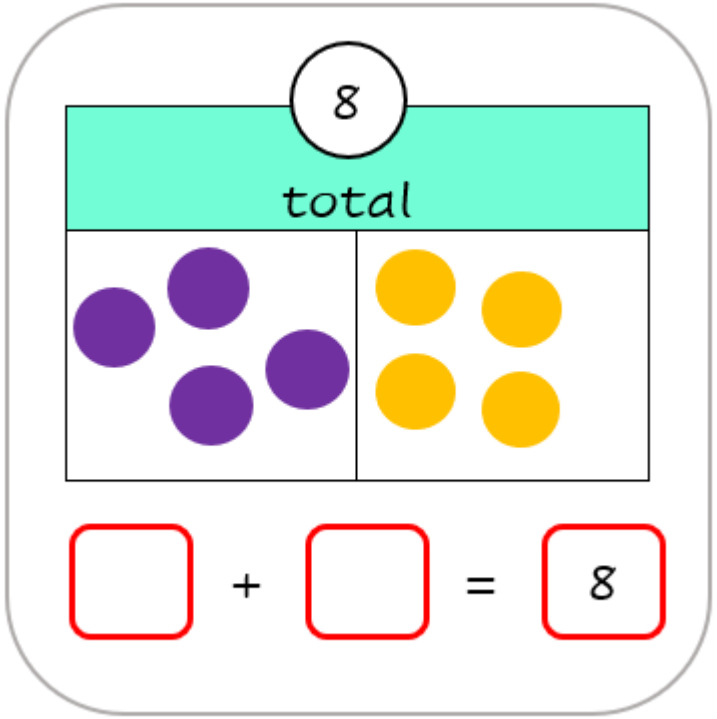






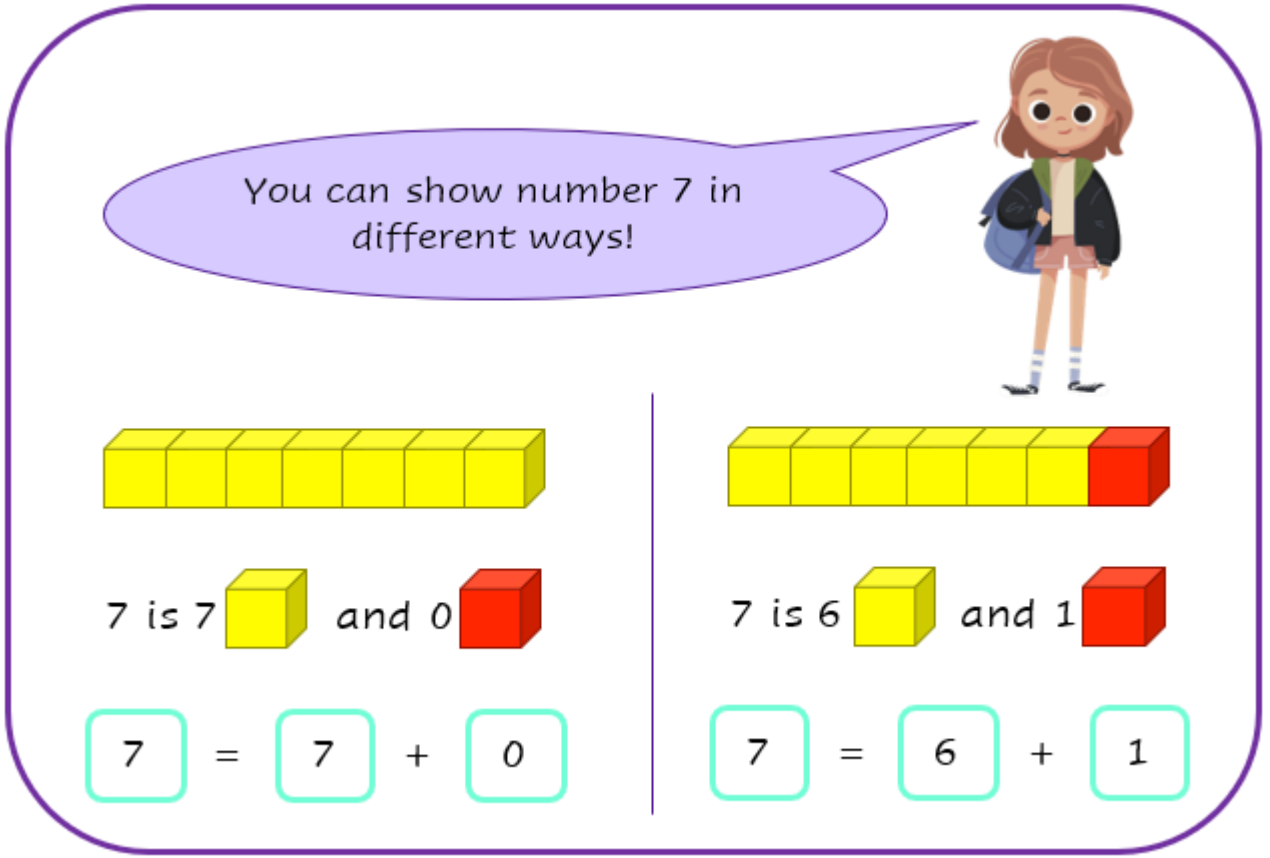
Composing and decomposing numbers (7)

- Another model used consists of a table that separate the whole into two parts.
- Students discover all the possible ways they can separate a number of cubes by placing part of the cubes in the left column and part of the cubes in the right column.
- They represent the table representation into a mathematical addition sentence.



Composing and decomposing numbers (8)

- Students notice all possible ways they can decompose a number in pictures and write corresponding mathematical addition sentences.



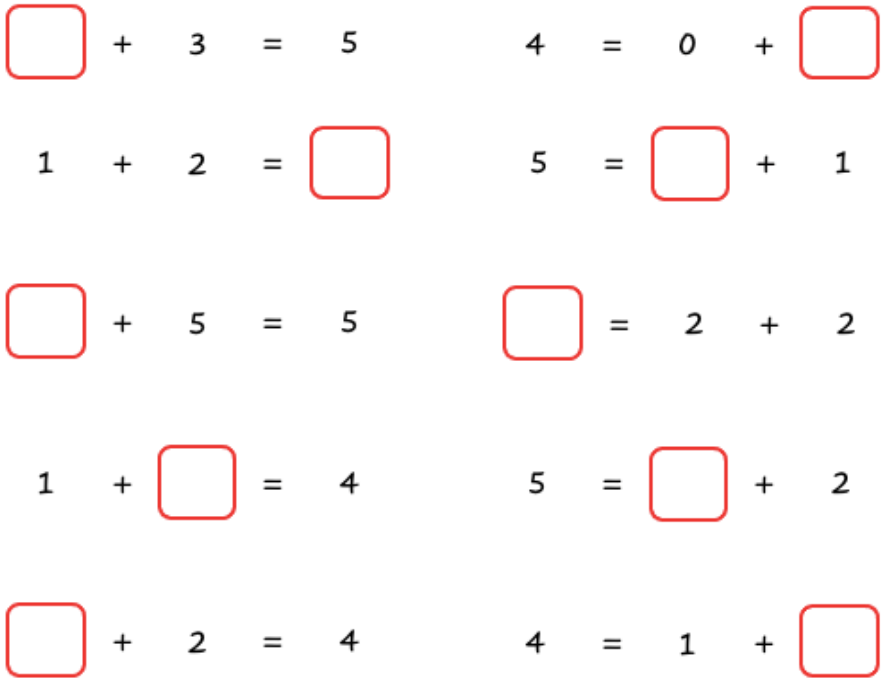
Addition and Subtraction Strategies

Addition and Subtraction Strategies

1. Addition Strategies – Composition and decomposition
2. Addition Strategies– Commutative property
3. Addition Strategies – Count on
4. Subtraction Strategies – Fact Families

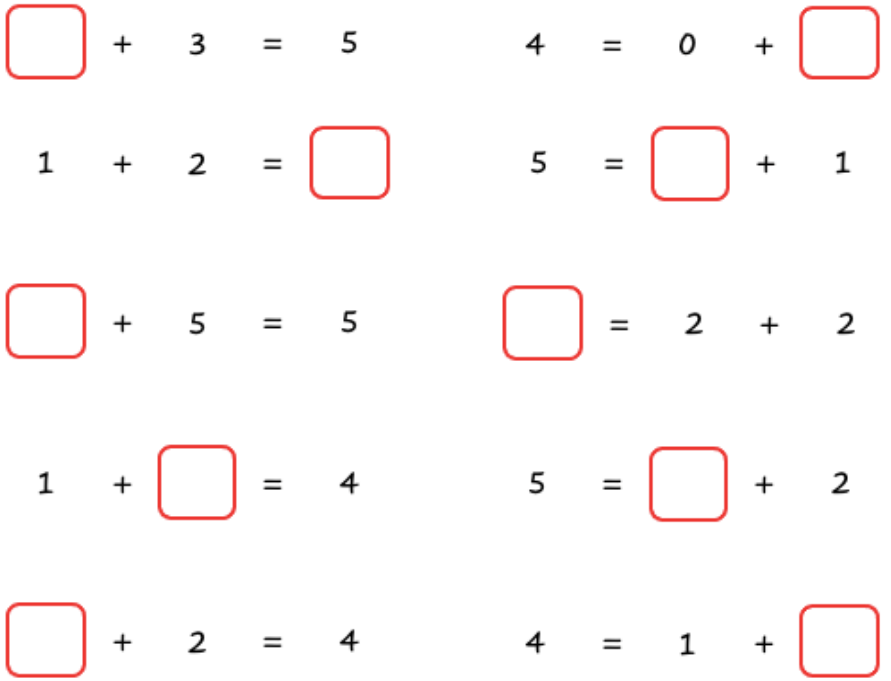
Addition Strategies- Composition and decomposition (1)

- We focus mainly on composing and decomposing numbers.
- Based on the pairs learned during the composition and decomposition of numbers, students are anticipated to find the missing number in the mathematical sentences.



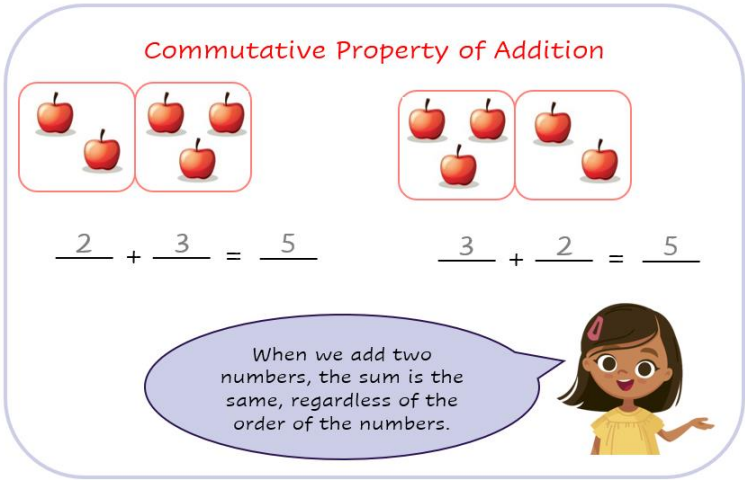
Addition Strategies- Composition and decomposition (2)

- We ask questions such as:
  - **“If I divide 5 into two parts and one part is 4, then what would the other part be?”**
  - **“What number should I add to 4 so that the sum is 5?”**



Addition Strategies– Commutative property

- To relieve students' memory, we introduce the commutative property of addition.
- The commutative property is presented at the concrete, pictorial and symbolic levels using multiple examples.



If 1 + 3 = 4, then 3 + 1 = ____

If 3 + 2 = 5, then 2 + 3 = ____

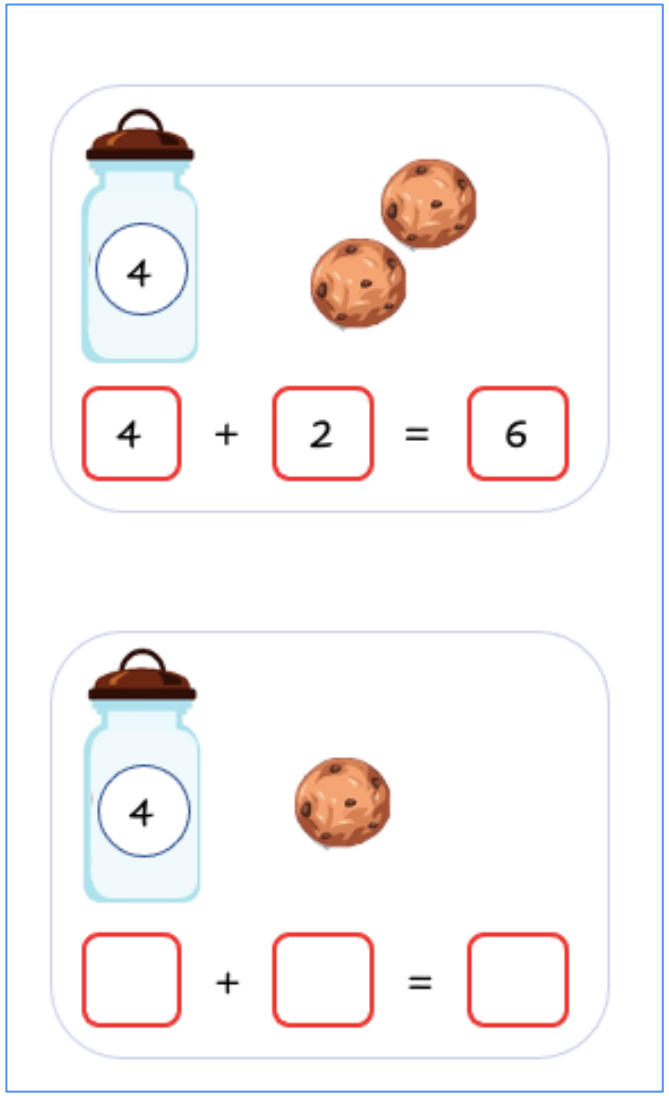
If 0 + 6 = 6, then 6 + 0 = ____

If 4 + 1 = 5, then 1 + 4 = ____

If 5 + 0 = 5, then 0 + 5 = ____

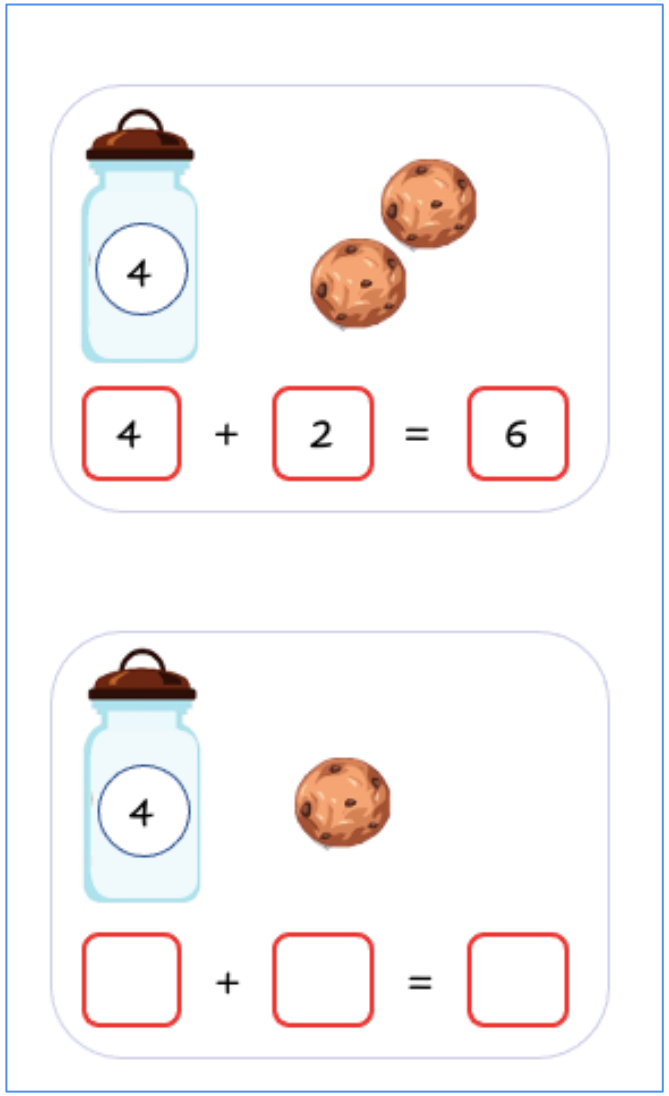
Addition Strategies– Counting up

- We introduce the "counting up" strategy to support students in selecting the larger addend and then start counting up the second addend.
- We present students with mathematical sentences, such as 8+1, to help them practise the counting up strategy.



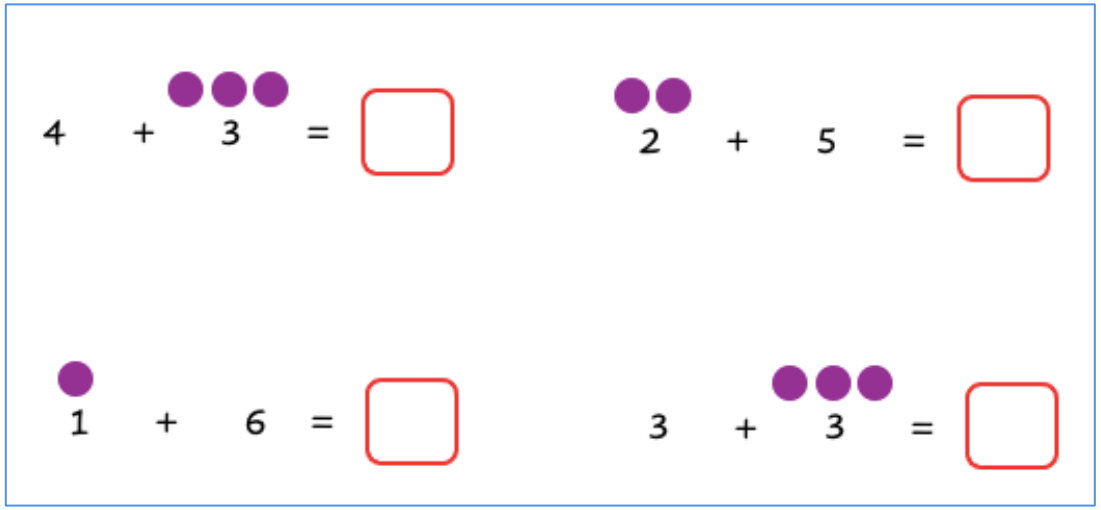
Addition Strategies– Counting up

- We present an opaque container and stick a number on the outer surface. Outside the container, we place discrete objects (real or pictures).
- Starting with the number written on the container, we count up according to the number of objects outside the container to find the total.



Addition Strategies– Counting up

- We then introduce mathematical addition sentences. We present the second addend quantitatively by marking a corresponding number of dots above the number.
- We urge the students to start with the first addend and count up based on the dots.
- We begin with the largest addend and count up.



Subtraction Strategies– Fact families

- Subtraction is introduced by making connections with addition.
- The relationship of addition and subtraction is presented again on a **concrete, pictorial a**nd **symbolic** level using multiple examples.

Write a number sentence for each picture to create a fact family.